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HF CHANNEL SIMULATOR FOR WIDEBAND SIGNALS. A MATHEMATICAL MODEL--ETC(U)

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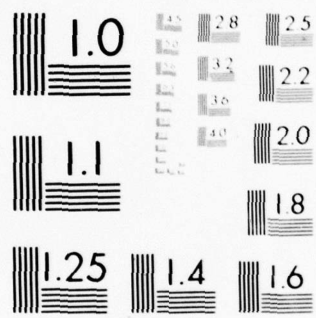

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HF CHANNEL SIMULATOR FOR WIDEBAND SIGNALS

A Mathematical Model and Computer Program for 100-kHz Bandwidth HF Channels

R Lugannani and HG Booker
LE Hoff (Contract Monitor)

31 March 1978

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A simulator has been developed that is capable of modeling the wideband HF channel. It is designed for carrier frequencies in the 2-32 MHz range and it can accommodate bandwidths up to 100 kHz and ground ranges up to 1000 km. It models the groundwave and ionosphere returns, but does not include additive noise or signal distortion produced in the transmitter or receiver. The inputs are the basic physical quantities that characterize the ionosphere (height, thickness, and maximum electron density of the E and F regions, sunspot number, time of day, etc) and those quantities that characterize the transmitter-receiver environment (carrier frequency, range, geographic location, antenna patterns, etc). Explicit values for channel parameters such as delay, attenuation, and doppler

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20. shift and spread are not required as these are calculated internally.

The simulator is divided into two parts. The first part is concerned with the mathematical model of the ionosphere and the computation of the channel parameters from the basic inputs. These channel parameters are used in the second part to determine the response for arbitrary inputs. A software version of the simulator has been implemented and tested, and a Fortran listing of the program is appended. The program runs in non-real time and requires a computer with a Fortran IV compiler and at least 28 K of core.

OBJECTIVE

Develop a mathematical model for the wideband HF channel and the software implementation of this model. The model must be valid for carrier frequencies in the 2-32 MHz range, for bandwidths up to 100 kHz, and for ground ranges up to 1000 km.

RESULTS

A simplified model of the ionosphere is employed to determine the various skywave returns and the effect these returns have on the overall received signal. The groundwave is modeled as a delayed, attenuated version of the transmitted signal. Inputs to the simulator consist of those basic physical quantities that characterize the ionosphere and those that characterize the transmitter-receiver environment; values of delay, attenuation, doppler shift and spread, and dispersion are calculated internally for each component of the received signal. A FORTRAN version of the simulator has been written and tested for several typical channels.

RECOMMENDATIONS

Make further refinements in the mathematical model in order to expand the scope and accuracy of the simulator. Of particular value in this regard would be the inclusion of a deviative absorption term for the skywave returns and the inclusion of an attenuation term for propagation of the groundwave over a rough, lossy ocean surface.

Investigate the relationship between the simulator inputs and the actual observed returns as measured, for example, by an ionosonde. A knowledge of this relationship would provide further verification of the mathematical model as well as a set of inputs that correspond to known operating conditions.

Finally, in view of the meager data base available in the wideband case, any measurement program aimed at enlarging this data base would represent a significant contribution to the study and modeling of HF channels.

GLOSSARY

English Letter Symbols

a	Multiplicative constant used to generate random sequence $\{Y_n\}$
$a_j(n)$	Impulse response of filter used to generate approximate random gain $\tilde{G}_j(nT_o)$
A	Multiplicative constant for doppler shift
A_g	Gain of groundwave
A_j	RMS gain of j-th ionosphere path
$A(x)$	$= 2\ G \tan \theta$
B	Multiplicative constant for doppler spread
B_r	Bearing of receiver as seen from transmitter in magnetic coordinates
BW	Two-sided bandwidth of transmitted signal
c	Velocity of light
$C_{G_j}(\tau)$	Correlation function of $G_j(t)$
$C_{\tilde{G}_j}(\tau)$	Correlation function of $\tilde{G}_j(t)$
$C_{U_j}(\tau)$	Correlation function of $U_j^{(1)}(t)$ and $U_j^{(2)}(t)$
D	Virtual path length
D_g	Delay of groundwave
$D_j^{(o)}$	$= f_c \cdot \tau_j^{(o)}$
$D_j^{(k)}$	$= \frac{1}{k!}\ [k\tau_j^{(k-1)} + f_c\tau_j^{(k)}]$, $k = 1,2,...$
f	Frequency
f_c	Carrier frequency

GLOSSARY (Continued)

f_r	Doppler reference frequency
f_{HD}	Gyrofrequency of D region
f_{HE}	Gyrofrequency of E region
f_{HF}	Gyrofrequency of F region
f_{pE}	General expression for penetration frequency of E region (ordinary or extraordinary wave)
f_{pF}	General expression for penetration frequency of F region (ordinary or extraordinary wave)
$f_{pE}^{(o)}$	Penetration frequency of E region - ordinary wave
$f_{pE}^{(x)}$	Penetration frequency of E region - extraordinary wave
$f_{pF}^{(o)}$	Penetration frequency of F region - ordinary wave
$f_{pF}^{(x)}$	Penetration frequency of F region - extraordinary wave
$G = G(f)$	Group height of signal path, oblique incidence
$G_j(t)$	Random gain of j-th path
$\tilde{G}_j(t)$	Approximation for random gain of j-th path
$G_R(\theta)$	Gain of receiving antenna
$G_T(\theta)$	Gain of transmitting antenna
$G_v = G_v(f)$	Group height of signal path, vertical incidence
h	Height above earth
h_D	Height of peak electron density of D region
h_E	Height of peak electron density of E region

GLOSSARY (Continued)

h_F	Height of peak electron density of F region
h_r	Height at which refractive index equals zero at vertical incidence
$h_j(t)$	Distortion impulse response of j-th path
$\hat{h}_j(t)$	Modified distortion impulse response: $=T_o h_j(t)$
$2H_E$	Semithickness of E region electron density
$2H_F$	Semithickness of F region electron density
$H_j(f)$	Fourier transform of $h_j(t)$
I	Angle of inclination of earth's magnetic field
I(x)	Cannonical expression representing general impulse response $\hat{h}_j(t)$
$I_p(x)$	Integrals used in series representation of I(x)
k	Sunspot number multiplier for non-deviative absorption loss
K	Multiplicative constant for non-deviative absorption loss
K_j	Constant used in the generation of $\tilde{G}_j(nT_o)$
L_{abs}	Absorption loss
L_{ant}	Antenna pattern loss
L_d	Spreading loss
L_j	Total loss of j-th path
M	Length of pseudo random sequence
N=N(h)	Electron density
N_E	Maximum electron density of E region

N_F	Maximum electron density of F region
N_g	Integer multiple of T_o nearest the groundwave delay D_g
N_j	Integer multiple of T_o nearest the signal delay D_j
$P_{G_j}(f)$	Power spectral density of $G_j(t)$
$P_{\tilde{G}_j}(f)$	Power spectral density of $\tilde{G}_j(t)$
$P_{U_j}(f)$	Power spectral density of $U_j^{(1)}(t)$ and $U_j^{(2)}(t)$
$r(t)$	Received, complex, baseband signal
R	Distance between transmitter and receiver
$s(t)$	Transmitted, complex, baseband signal
$\tilde{s}_j(t)$	Distorted, complex signal returned from ionosphere via j-th path
$\hat{s}_j(t)$	Distorted, complex signal returned from ionosphere via j-th path after removal of phase and delay components: $\tilde{s}_j(t) = \exp[-2\pi i D_j^{(o)}]\ \hat{s}_j(t-D_j^{(1)})$
S	Sunspot number
$S(f)$	Fourier transform of $s(t)$
t	Time
T_o	Sampling interval (one half the Nyquist interval of the transmitted signal)
u_1, u_2	Uniform [0,1] random variables
$U_j^{(1)}(t), U_j^{(2)}(t)$	Real, independent, zero mean identically-distributed, Gaussian process used to define $G_j(t)$
$W_j(1), W_j(2), W_j(3)$	Independent, complex, Gaussian random variables used in generation of $\tilde{G}_j(nT_o)$
x	$= \cos\theta$
$\{Y_n\}$	Sequence of pseudo random integers

GLOSSARY (Continued)

Greek Letter Symbols

α	Doppler shift exponent
β	Doppler spread exponent
γ	Exponent of solar zenith angle term in expression for nondeviative absorption
$\gamma_j(1)$, $\gamma_j(2)$	Constants used in the generation of $\tilde{G}_j(nT_o)$
$\Delta^{(2)}$	General amplitude distortion coefficient
$\Delta^{(3)}$	General phase distortion coefficient
$\Delta_j^{(2)}$	Normalized amplitude distortion coefficient of j-th path
$\Delta_j^{(3)}$	Normalized phase distortion coefficient of j-th path
η	Normalized frequency variable = f/BW
θ	Ray angle of signal path measured from vertical
κ	$= (e^2/4\pi^2\varepsilon_o m) \approx 80.5$
λ	Geographic longitude
λ_o	Geographic longitude of north magnetic pole
$\lambda_j(1)$, $\lambda_j(2)$	Constants used in the generation of $\tilde{G}_j(nT_o)$
μ	Refractive index
ν	General doppler shift
ν_j	Doppler shift of j-th path
ν_r	Reference doppler shift

GLOSSARY (Continued)

ξ	Complex Gaussian random variable
$\xi_j(n)$	Independent, identically-distributed Gaussian sequence
$\xi_j^{(1)}(n)$	Real part of $\xi_j(n)$
$\xi_j^{(2)}(n)$	Imaginary part of $\xi_j(n)$
ρ	Parameter used in fitting $C_{\tilde{G}_j}(\tau)$ to $C_{G_j}(\tau)$
2σ	General doppler spread
$2\sigma_j$	Doppler spread of j-th path
$2\sigma_r$	Reference doppler spread
τ	General path delay
$\tau_j(f)$	Delay of j-th path as a function of frequency
$\tau_j^{(k)}$	k-th derivative of delay of j-th path evaluated at the carrier frequency f_c
ϕ	Geographic latitude
ϕ_o	Geographic latitude of north magnetic pole
Φ	Magnetic latitude
χ	Solar zenith angle
ψ_i	Angle between earth's magnetic field and wave incident upon ionosphere
ψ_r	Angle between earth's magnetic field and wave reflected from ionosphere

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I. INTRODUCTION

The use of channel simulators to study the performance of communication systems and to assist in their design has become increasingly popular in recent years. By employing a simulator, it is possible to reproduce given channel characteristics as desired and to compare different system designs in identical environments. Moreover, the cost of simulation is usually much less than the cost of "on the air" measurements and it is quite reliable. A variety of HF channel simulators have been proposed and studied in the past [1-4]*. Sailors and Hill [5] have written an excellent survey of the field and have made several recommendations for future work. Some of these recommendations are embodied in the present report.

In the following the development of a simulator for the wideband HF channel is described. This simulator is designed for carrier frequencies in the 2-32 MHz range and it can accommodate bandwidths up to 100 kHz and ground ranges up to 1000 km. The simulator models the groundwave and all ionosphere returns (both single and multiple hops), but does not include additive noise or signal distortion produced in the transmitter or receiver. It takes into account all ionospheric phenomena that have a significant effect on wideband communications. Many of these phenomena, such as delay differences between the ordinary and extraordinary waves and signal distortion caused by variations of delay with frequency, have been neglected in the earlier simulators, resulting in severe bandwidth limitations. The inputs required are the basic parameters that characterize the physical ionosphere (height, thickness, and maximum electron density of the E and F regions, sunspot number, time of day, etc.) and those that characterize the transmitter and receiver (carrier frequency, range, geographic location, antenna patterns, etc.). Since these input parameters are the ones that are actually known in practice it is a simple matter to vary them and observe the effect they have on system performance. This

*References are listed in Section VI.

capability does not exist in the previous simulators which require as inputs not the basic quantities mentioned above but those channel parameters such as delay, attenuation, doppler shift and doppler spread derived from them.

In the course of developing the simulator a number of simplifications and approximations have been made to obtain tractable results. The most important of these assumptions concerns the electron density profile and the absorption mechanism. It is recognized that the ionosphere model used does not provide a detailed physical characterization of the ionosphere; however, it does provide an "order of magnitude" characterization suitable for communication purposes. The simulator is divided into two parts. The first part is concerned with the mathematical model of the ionosphere and the computation of the channel parameters from the basic inputs. The second part uses these parameters to determine the channel response for arbitrary inputs. Both parts of the simulator are described in detail in the following sections.

II. THE HF CHANNEL

The signal received via the HF channels considered in this report is the sum of the groundwave and the various ionosphere returns. Throughout we shall assume that the groundwave is an attenuated, delayed version of the transmitted signal but is otherwise undistorted. The ionosphere returns also suffer attenuation and delay but in addition are subject to fading and distortion. There is also time dispersion (multipath) produced by the different delays associated with each ionosphere return. The purpose of this section is to present some definitions that are used in the sequel and to introduce the mathematical model for the HF channel.

The complex transmitted baseband signal will be denoted by $s(t)$ and its Fourier transform by $S(f)$. Thus

$$s(t) = \int_{-\infty}^{\infty} S(f) \exp [2\pi i f t] df. \tag{1}$$

Distorted versions of this signal will be returned from the ionosphere for both the E region and the F region (low ray and high ray). There will be a set of these returns for the ordinary and extraordinary magnetoionic components and for as many multiple-hop paths as are present. Although many returns are possible, all but a few of these experience a large attenuation and the number of effective returns is usually small. In the following we will denote the number of effective ionosphere returns by N. This number is determined by excluding all returns that suffer an attenuation greater than some predetermined value.

Distortion of signals reflected from the ionosphere has been studied by several authors, who have attempted to characterize the impulse response for a simplified model of the ionosphere [6-10]. Unfortunately, these results have been derived using ionosphere

models that are too idealized for our purposes[1] and the impulse responses obtained are too complicated for easy implementation[2]. In order to obtain tractable expressions for the distortion we shall assume that it is caused primarily by the fact that the delay is a function of frequency and is not constant across the signal bandwidth. This characterization of the distortion mechanism allows us to make a number of approximations which result in relatively simple expressions for the distorted signal. For obvious reasons we shall refer to this particular distortion as delay distortion.

Let f_c denote the carrier frequency and let $\tau_j(f)$ denote the delay of the j-th return as a function of frequency. It follows from (1) that the distorted baseband signal returned from the ionosphere is given by

$$\tilde{s}_j(t) = \int_{-\infty}^{\infty} S(f) \exp\left[-2\pi i(f_c+f)\cdot\tau_j(f_c+f)\right]\cdot\exp\left[2\pi ift\right] df. \qquad j = 1,\cdots,N \tag{2}$$

We will characterize this distortion by the coefficients in the power series expansion of the delay about the carrier frequency. This series appears as

$$\tau_j(f_c+f) = \sum_{k=0}^{\infty} \frac{f^k}{k!} \tau_j^{(k)}, \; j=1,\ldots,N \tag{3}$$

---

[1] They allow for reflection only from a single ionospheric region and consider such simple electron density profiles as a constant and a linear function of height.

[2] In one case the impulse response is given as the solution of an integral equation.

with

$$\tau_j^{(k)} = \frac{d^k}{df^k} \tau_j(f_c+f)\Big|_{f=0} . \tag{4}$$

Substituting this series into the exponential delay term in (2) we obtain

$$\exp[-2\pi i(f_c+f)\cdot\tau_j(f_c+f)] = \exp\left[-2\pi i \sum_{k=0}^{\infty} f^k D_j^{(k)}\right] \qquad j=1,\cdots,N \tag{5}$$

where we have defined

$$D_j^{(o)} = f_c\tau_j^{(o)} \qquad j=1,\cdots N \tag{6}$$

and

$$D_j^{(k)} = \frac{1}{k!}\left[k\tau_j^{(k-1)} + f_c\tau_j^{(k)}\right] \qquad k=1,2,\cdots \quad j=1,\cdots N. \tag{7}$$

Thus, for $\tilde{s}_j(t)$ we have

$$\tilde{s}_j(t) = \exp\left[-2\pi i D_j^{(o)}\right] \int_{-\infty}^{\infty} S(f)\exp\left[-2\pi i \sum_{k=2}^{\infty} f^k D_j^{(k)}\right]$$

$$\cdot\exp\left[2\pi i f(t-D_j^{(1)})\right] df. \qquad j=1,\cdots,N \tag{8}$$

For computational reasons it is convenient to separate the phase and delay terms from the actual distortion of the signal. To this end we define a function $\hat{s}_j(t)$ as follows

$$\tilde{s}_j(t) = \exp[-2\pi i D_j^{(o)}]\ \hat{s}_j(t-D_j^{(1)}) \qquad j=1,\cdots N \tag{9}$$

where

$$\hat{s}_j(t) = \int_{-\infty}^{\infty} S(f)H_j(f)\exp[2\pi ift]dt \qquad (10)$$

$$j=1,\cdots,N$$

with

$$H_j(f) = \exp\left[-2\pi i\sum_{k=2}^{\infty} f^k D_j^{(k)}\right], \ j=1,\cdots N. \qquad (11)$$

In practice it is convenient to determine $\hat{s}_j(t)$ by performing a time domain convolution rather than by evaluating the Fourier transform (10). If the impulse response determined by $H_j(f)$ is denoted by $h_j(t)$, we have

$$h_j(t) = \int_{-\infty}^{\infty} H_j(f)\exp[2\pi ift]df, j=1,\cdots N., \qquad (12)$$

and the desired convolution appears as

$$\hat{s}_j(t) = \int_{-\infty}^{\infty} s(t-u)h_j(u)du, \ j=1,\cdots N. \qquad (13)$$

A discussion of the approximations used in the numerical evaluation of $h_j(t)$ and $\hat{s}_j(t)$ is presented in Section IV.

In addition to the above distortion, the ionosphere returns are subject to attenuation and random fading. These two phenomena are included by multiplying the delayed, distorted signal by a random gain, which we denote by $G_j(t)$. We will assume that $G_j(t)$ is a complex, zero-mean, Gaussian process with identically-distributed real and imaginary parts. This is the Rayleigh fading model [11]. While the assumption of Rician fading [12] would provide somewhat more flexibility than the Rayleigh assumption, the available evidence [12-14] indicates that the Rayleigh model is adequate for our purposes. The random gains associated with different returns are assumed to be independent.

Let $U^{(1)}_j(t)$ and $U^{(2)}_j(t)$ denote two real, independent zero mean, identically-distributed, stationary Gaussian processes with correlation function

$$C_{U_j}(\tau) = E\, U^{(1)}_j(t+\tau)U^{(1)}_j(t) = E\, U^{(2)}_j(t+\tau)U^{(2)}_j(t). \qquad (14)$$

$$j=1,\cdots N.$$

The corresponding power spectral density is given by

$$P_{U_j}(f) = \int_{-\infty}^{\infty} C_{U_j}(\tau)\ \exp[-2\pi i f\tau]d\tau, j=1,\cdots,N. \qquad (15)$$

We define the random gain in terms of $U_j^{(1)}(t)$ and $U_j^{(2)}(t)$ as follows

$$G_j(t) = [U_j^{(1)}(t) + i\ U_j^{(2)}(t)]\cdot \exp[2\pi i\nu_j t] \qquad (16)$$

$$j=1,\cdots N.$$

where ν_j is the doppler shift for the j-th ionosphere return. The correlation and power spectral density of $G_j(t)$ are easily determined using the assumed properties of $U_j^{(1)}$ and $U_j^{(2)}(t)$. Thus

$$C_{G_j}(\tau) = EG_j(t+\tau)G_j^*(t) = 2C_{U_j}(\tau)\cdot \exp[2\pi i\nu_j\tau] \qquad (17)$$

$$j=1,\cdots N.$$

and

$$P_{G_j}(f) = 2P_{U_j}(f-\nu_j)\ ,\ j=1,\cdots N. \qquad (18)$$

Following Watterson [1] we assume that the power spectral density $P_{G_j}(f)$ has a Gaussian shape. While there is no conclusive experimental evidence for this assumption, it is a convenient model for spectra that are unimodal and that decrease rapidly. Specifically we let

$$C_{G_j}(\tau) = A_j^2 \exp[-2\pi^2\sigma_j^2\tau^2 + 2\pi i\nu_j\tau] \quad . \qquad j=1,\cdots N \tag{19}$$

Here A_j is the rms gain ($0 \leq A_j \leq 1$) of the j-th return and $2\sigma_j$ is the doppler spread of the return. The power spectral density associated with this correlation function is

$$P_{G_j}(f) = \frac{A_j^2}{\sqrt{2\pi}\ \sigma_j} \exp\left[-\frac{(f-\nu_j)^2}{2\sigma_j^2}\right] \quad , \ j=1,\cdots N. \tag{20}$$

Finally, we must add the groundwave to the ionosphere returns. The groundwave is assumed to experience only delay and to have a non-random gain. If A_g denotes the gain ($0 \leq A_g \leq 1$) and D_g denotes the delay, the contribution to the received signal from the groundwave is given by $A_g\ s(t-D_g)$. The gain is a complicated function of range, sea state and frequency. Graphs of this functional relationship are given in [15].

Summarizing the above, the complex, baseband received signal can be expressed as

$$r(t) = A_g\ s(t-D_g) + \sum_{j=1}^{N} \exp[-2\pi i D_j^{(o)}]\ G_j(t)\ \hat{s}_j(t-D_j^{(1)}). \tag{21}$$

A block diagram illustrating (21) is presented in Figure 1.

In the following sections, the quantities needed in the evaluation of r(t) are derived and the techniques employed in its numerical evaluation are described.

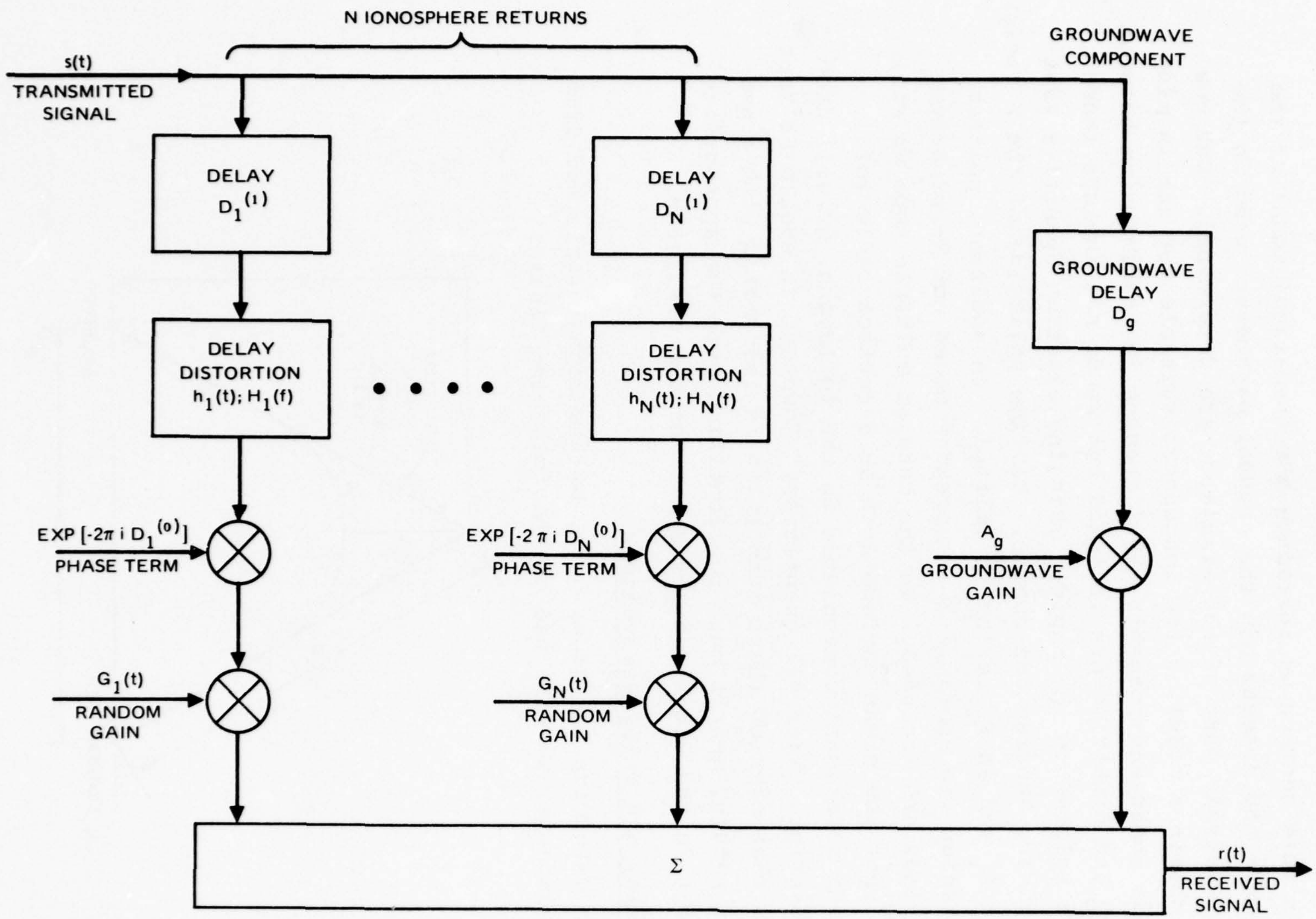

Figure 1. Block Diagram of Simulator

III. MATHEMATICAL MODEL OF THE IONOSPHERE

In this section we introduce a mathematical model of the ionosphere and discuss how the channel parameters used in the signal analysis part of the simulator are determined. Our goal is to obtain a model of the ionosphere suitable for use in mid-latitude ocean environments. Differences in transmission between daytime and nighttime are included but we do not include those effects related to the rapidly changing electron densities that occur during sunrise and sunset. Daytime splitting of the F region into an F_1 and an F_2 region is omitted. In addition, neither transmission via sporadic E or spread F modes nor M- or N-type reflections are included. In the case of multiple hops we will assume that the ocean surface acts as a perfect reflector.

As a notational convenience in the following, we will drop the subscript j from all quantities. However, it should be kept in mind that each of these quantities is associated with a particular return, and it must be identified with the appropriate subscript in the signal analysis part of the simulator.

A. Delay and Delay Derivatives

To determine the group delay of the signal reflected from the ionosphere, we make use of the following figure.

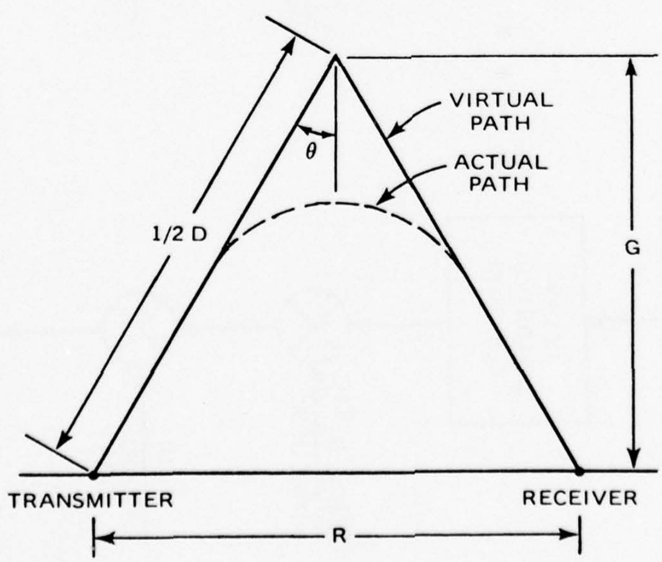

Figure 2. Virtual and Actual Signal Paths

Here the dashed line is the actual ionospheric path of the signal between the transmitter and the receiver and the solid line is the virtual path. The ground range between transmitter and receiver is denoted by R, the virtual path length by D, the ray angle measured from the vertical by θ and the group height by G. Assuming a flat earth and ionosphere, we have by simple geometry

$$D = \frac{R}{\sin\theta} = \frac{R}{(1-x^2)^{\frac{1}{2}}} \tag{22}$$

where we have defined $x = \cos\theta$. If c denotes the velocity of light, the signal delay is given by the virtual path length divided by c

$$\tau = \frac{R}{c(1-x^2)^{\frac{1}{2}}} . \tag{23}$$

It remains to determine the ray angle θ.

For vertical incidence (θ=0), the group height is given by the integral [16]

$$G_v = G_v(f) = \int_o^{h_r} \frac{dh}{\mu} \tag{24}$$

where μ is the refractive index and h_r is the height at which μ=0. In the absence of both collisions and the earth's magnetic field, the refractive index is given by [16]

$$\mu^2 = (1-\kappa \frac{N}{f^2}) \tag{25}$$

where N = N(h) is the electron density in m^{-3}, f is the frequency in Hz and $\kappa = (e^2/4\pi^2\varepsilon_o m) \approx 80.5$. The group height for oblique incidence is the same as for vertical incidence but we must replace f by f cos θ [16]. That is $G = G(f) = G_v(f \cos\theta)$. Once the group height has been determined, we appeal to Figure 2 to obtain the relationship

$$R = 2\ G \tan\theta = 2\ G \left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}} . \tag{26}$$

Defining

$$A(x) = 2\,G\left(\frac{1-x^2}{x^2}\right)^{1/2} \tag{27}$$

we see that, for a specified range R, we must solve the transcendental equation

$$R = A(x) \tag{28}$$

for x and then substitute this into (23) to obtain the delay.

We seek an approximation to the electron density for which the group height, G, can be expressed in a simple closed form. Our main concern here is with reflections from the E and F regions. At nighttime we assume that the electron density profile can be represented by two distinct parabolas as shown in Figure 3. Let N_E be the peak electron density of the E region in m^{-3}, let h_E be the height of the peak electron density of the E region in km, and let $2H_E$ be the semithickness of the E region in km; similar definitions are made for the F region. Then the nighttime electron density is given by

$$N(h) = \begin{cases} 0 & h < h_E - 2H_E \\ N_E\left[1-\left(\frac{h-h_E}{2H_E}\right)^2\right], & h_E-2H_E \le h \le h_E+2H_E \\ 0\ , & h_E+2H_E < h < h_F-2H_F \\ N_F\left[1-\left(\frac{h-h_F}{2H_F}\right)^2\right], & h_F-2H_F \le h \le h_F+2H_F \\ 0 & , \quad h_F+2H_F < h\ . \end{cases} \tag{29}$$

In the daytime the electron density profile is also assumed to be parabolic but allowance is made for ionization between the E and F regions. The electron density between the two regions is assumed to be constant and equal to the peak value of the E region. This is illustrated in Figure 4. Using the same definitions as before, the daytime electron density is given by

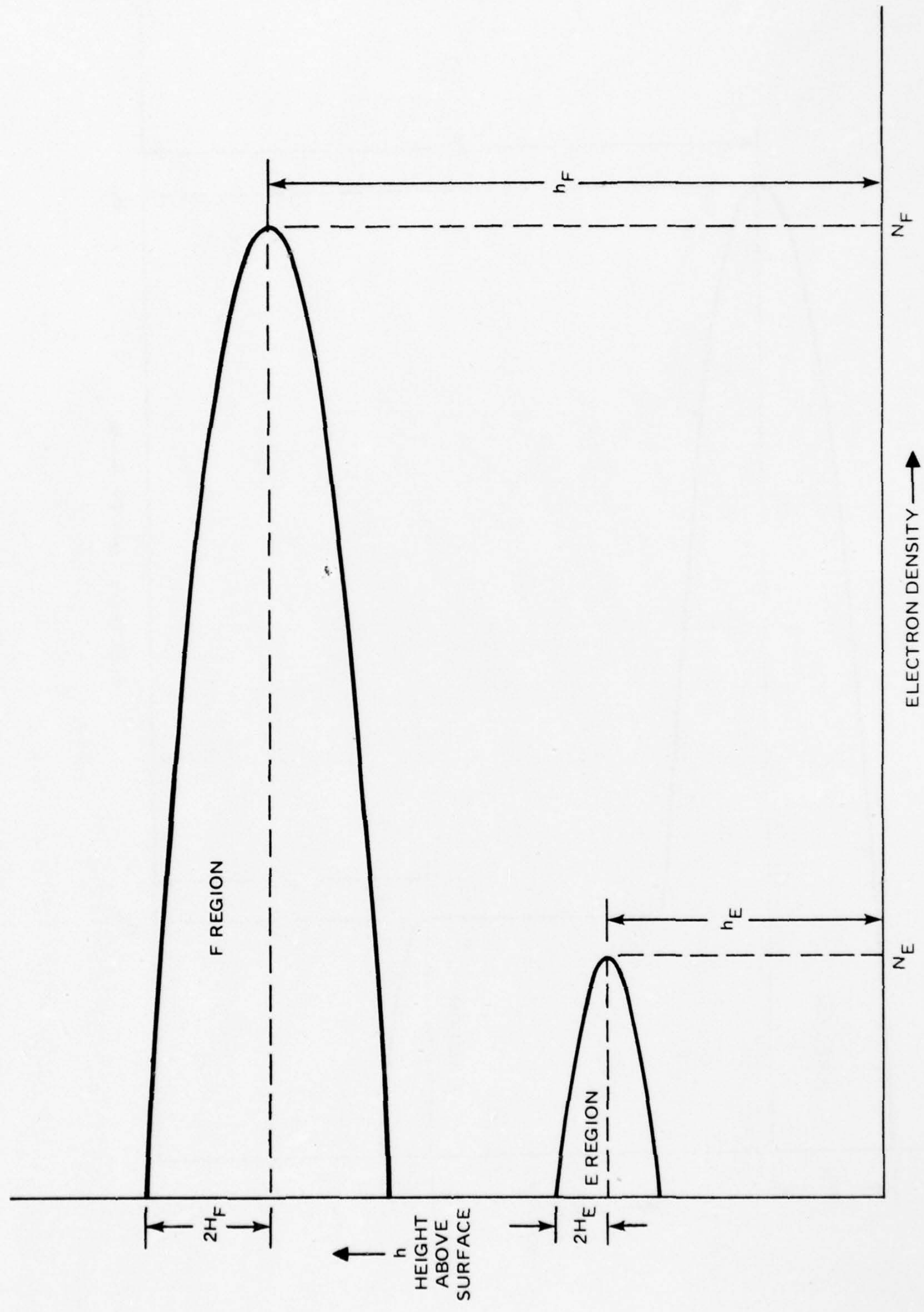

Figure 3. Nighttime Electron Density Profile

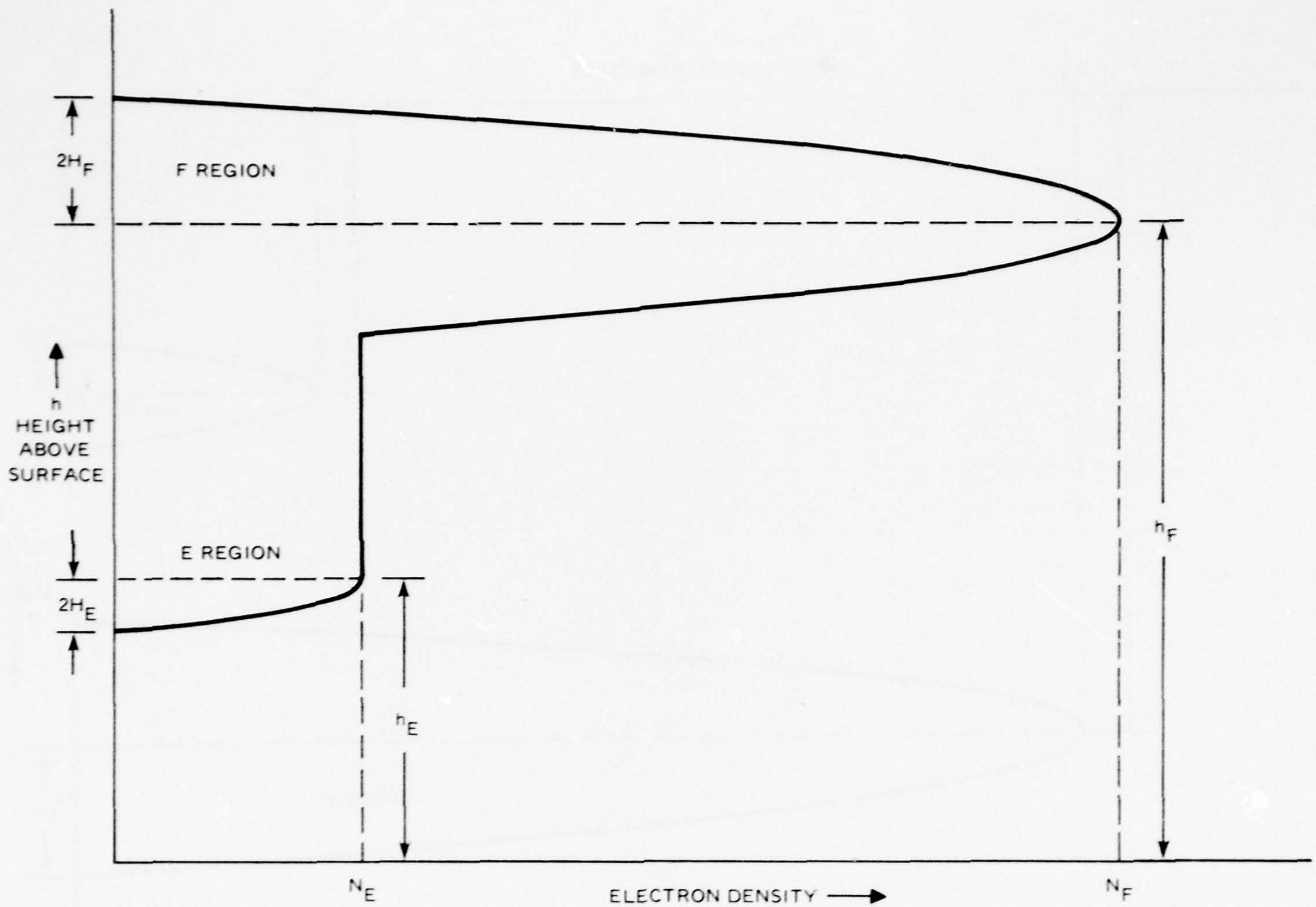

Figure 4. Daytime Electron Density Profile

$$N(h) = \begin{cases} 0\ , & h<h_E-2H_E \\ N_E\left[1-\left(\dfrac{h-h_E}{2h_E}\right)^2\right], & h-2H_E \le h \le h_E \\ N_E\ , & h_E<h<h_F+2H_F\left(1-\dfrac{N_E}{N_F}\right)^{\frac{1}{2}} \\ N_F\left[1-\left(\dfrac{h-h_F}{2h_F}\right)^2\right], & h_F+2H_F\left(1-\dfrac{N_E}{N_F}\right)^{\frac{1}{2}} \le h \le h_F+2H_F \\ 0\ , & h_F+2H_F<h. \end{cases} \tag{30}$$

Note that the constants appearing in (24) and (30) will be different owing to the different heights and ionization densities for nighttime and daytime conditions.

A quantity of interest in the present analysis is the penetration frequency of each region. This is defined as the plasma frequency at the peak electron density [16]. We have for the E and F regions

$$f_{pE}^{\ 2} = \kappa N_E \tag{31a}$$

$$f_{pF}^{\ 2} = \kappa N_F \tag{31b}$$

where $\kappa = (e^2/4\pi^2\varepsilon_o m) \simeq 80.5$ and the two penetration frequencies are in Hz. The physical interpretation of f_{pE} and f_{pF} is that they represent the maximum frequencies at which signals will be reflected from the respective regions at vertical incidence.

Returning to the determination of the group height, G, it is a relatively simple matter to evaluate the integral (24) for either of the above electron densities [17]. The details of this evaluation are presented in Appendix A.

In the foregoing the effect of the earth's magnetic field has been neglected, but we take this to be an adequate approximation for the ordinary wave. For the extraordinary wave, however, the penetration frequencies are shifted relative to those for the ordinary wave, and it is essential to take this into account.

Denoting the penetration frequencies of the ordinary and extraordinary waves by the superscripts "o" and "x" we have [16]

$$f_{pE}^{(x)} = \frac{1}{2} f_{HE} + \left[f_{HE}^{\ 2} + 4 \left(f_{pE}^{(o)}\right)^2 \right]^{\frac{1}{2}} \tag{32a}$$

$$f_{pF}^{(x)} = \frac{1}{2} f_{HF} + \left[f_{HF}^{\ 2} + 4 \left(f_{pF}^{(o)}\right)^2 \right]^{\frac{1}{2}} \quad . \tag{32b}$$

Here $f_{pE}^{(o)}$ and $f_{pF}^{(o)}$ are defined by (31), and f_{HE} and f_{HF} are the gyrofrequencies associated with the earth's magnetic field at the levels of maximum ionization density in the E and F regions. For the gyrofrequencies in mHz we will use the approximate expressions [16]

$$f_{HE} = 0.87 \left(\frac{6370}{6370 + h_E}\right)^3 (1 + 3 \sin^2 \Phi)^{\frac{1}{2}} \tag{33a}$$

$$f_{HF} = 0.87 \left(\frac{6370}{6370 + h_F}\right)^3 (1 + 3 \sin^2 \Phi)^{\frac{1}{2}} . \tag{33b}$$

where Φ is the latitude expressed in magnetic coordinates. If ϕ and λ are respectively the geographic latitude and longitude, and ϕ_o and λ_o represent the latitude and longitude of the north magnetic pole ($\phi_o \approx 78.3°N$, $\lambda_o \approx 69°W$), then [16]

$$\sin \Phi = \sin \phi \sin \phi_o + \cos \phi \cos \phi_o \cos (\lambda - \lambda_o) \quad . \tag{34}$$

We assume that, to calculate delays for the extraordinary wave, we may use the expressions in Appendix A but with $f_{pE}^{(x)}$ and $f_{pF}^{(x)}$ substituted for the ordinary wave penetration frequencies. Otherwise the calculations of delay are assumed to take the same form as those for the ordinary wave.

To characterize delay distortion we need the derivatives of the delay evaluated at the carrier frequency. Actually, only the first three derivatives are required for the approximation discussed in Section IV. The determination of these three derivatives is straightforward although the resulting expressions are quite complicated. They are listed in Appendix A.

For multiple hop transmissions we assume that the ionosphere parameters are identical for each hop. In the two-hop case the foregoing analysis is applied with the original range multiplied by 1/2. The angle is then correct as calculated but the path length, delay and delay derivatives must be multiplied by 2. Higher numbers of hops are treated in a similar manner using appropriate multipliers.

B. Attenuation

Three main sources of attenuation are included in the present model. They are: (1) nondeviative absorption, (2) antenna gain, and (3) spreading ($1/r^2$ loss). Other factors such as focusing and polarization mismatch will also affect the received signal strength, but these are assumed to be of secondary importance and they have accordingly been omitted.

B.1 Absorption

Absorption is assumed to take place only in the D region of the ionosphere and only in the daytime. The absorption is assumed to be nondeviative, i.e., it takes place in a region where the refractive index is near unity. The attenuation will be characterized by the expression [16]

$$L_{abs} = K \sec \theta \cdot (1+kS) \cdot (\cos \chi)^{\gamma} \tag{35}$$
$$\cdot \left\{ \frac{1}{(f \pm f_{HD} |\cos \psi_i|)^2} + \frac{1}{(f \pm f_{HD} |\cos \psi_r|)^2} \right\} .$$

Here L_{abs} is the attenuation due to absorption in decibels per hop, θ is the ray angle measured from the vertical, S is the sunspot number, χ is the solar zenith angle, f is the frequency in mHz, f_{HD} is the gyrofrequency at the D region level in mHz and ψ_i [resp., ψ_r] is the angle between the earth's magnetic field and the incident [resp., reflected] wave. The plus sign is used for the ordinary wave and the minus sign is used for the extraordinary wave.

The relative importance of the sunspot number and solar zenith angle depends upon the constants K, k and γ; following Davies [16, p. 235] we will set K = 215, k = 0.0035 and γ = 0.75. The gyrofrequency, f_{HD}, is given by [16]

$$f_{HD} = 0.87 \left(\frac{6370}{6370+h_D} \right)^3 (1+3 \sin^2 \Phi)^{\frac{1}{2}} . \tag{36}$$

where h_D is the height of the D region in km and Φ is the latitude in magnetic coordinates. In this expression we will take h_D to be 70 km and use (34) to determine $\sin \Phi$. The angles ψ_i and ψ_r are determined from the relationships

$$\cos \psi_i = \sin \theta \cos B_r \cos I + \cos \theta \sin I \tag{37a}$$

$$\cos \psi_r = \sin \theta \cos B_r \cos I - \cos \theta \sin I \tag{37b}$$

where B_r is the bearing of the receiver as seen from the transmitter in magnetic coordinates and I, the angle of inclination of the earth's magnetic field, is given by

$$\tan I = -2 \tan \Phi. \quad (38)$$

For multiple-hop paths, the expression (35) must be multiplied by the number of hops.

B.2 Antenna Gain

No attempt has been made in this model to provide a variety of antenna patterns. While any desired antenna gain can be easily included in the program, we shall use the gain of a short vertical dipole for both transmitter and receiver. If G_T and G_R denote the transmitter and receiver power gains relative to isotropic antennas we have

$$G_T(\theta) = G_R(\theta) = \frac{3}{2} \sin^2 \theta . \quad (39)$$

Thus for the overall attenuation in decibels we have

$$L_{ant} = -10 \log_{10} G_T(\theta) - 10 \log_{10} G_R(\theta) = -20 \log_{10}[\frac{3}{2} \sin^2 \theta] . \quad (40)$$

B.3 Spreading Loss

The decrease in signal strength attributable to the length of the transmission path is given by

$$L_d = 20 \log_{10} \left(\frac{4\pi Df}{0.3}\right) \quad (41)$$

where L_d is the path loss in decibels, D is the total path length in km, taking into account any multiple hops, and f is the frequency in mHz.

The total attenuation for a particular ionosphere return is obtained by adding (35), (40) and (41), after making any necessary adjustments for multiple hops. Denoting the total attenuation for the j-th return by L_j we have

$$L_j = L_{abs} \times (\text{No. of Hops}) + L_{ant} + L_d . \quad (42)$$

The rms gain A_j is thus given by

$$A_j = 10^{-(L_j/20)} \ . \tag{43}$$

C. Doppler

The remaining quantities needed to characterize the random gains, $G_j(t)$, are the doppler shifts and the doppler spreads. In this report we are not concerned with the relatively large doppler values that occur during sunrise and sunset, but with the smaller values that are typical of the ionosphere for the majority of a typical twenty-four hour period. What is desired is the shift and spread associated with each of the propagation modes. Unfortunately, measurements that have been made concerning doppler values usually have not been concerned with the shift and spread of the individual propagation modes; instead they have recorded composite doppler values involving many modes [18]. Moreover, most theoretical studies have not been concerned with communication system performance and are of marginal value for present purposes [19-22].

What is required for our model is an expression relating the doppler shifts and spreads to frequency for each ionosphere return. This expression will depend on the electron density profile and its time rate of change as well as on various other ionosphere parameters [23]. We assume that the doppler values can be characterized adequately by the following power laws

$$\nu = A\ f^{\alpha} \tag{44}$$

and

$$\sigma = B\ f^{\beta} \tag{45}$$

where ν and 2σ are the doppler shift and doppler spread; f is the frequency in mHz; and A, B, α, and β are constants that characterize the region from which the wave is reflected and the magnetoionic

component involved. In general we need an expression of the above type for the low and high ray returns from each region and for each magnetoionic component.

It is convenient to express the constants A and B appearing in (44) and (45) in terms of a reference value measured at a known frequency. Thus the above expressions can be rewritten as

$$\nu = \nu_r (f/f_r)^{\alpha} \tag{46}$$

$$\sigma = \sigma_r (f/f_r)^{\beta} \tag{47}$$

where ν_r and $2\sigma_r$ are the doppler shift and spread measured at the reference frequency f_r. For multiple hops we assume that the doppler effects combine incoherently so that

$$\nu = (\text{No. of Hops}) \cdot \nu_r \cdot (f/f_r)^{\alpha} \tag{48}$$

and

$$\sigma = (\text{No. of Hops})^{\frac{1}{2}} \cdot \sigma_r \cdot (f/f_r)^{\beta} \quad . \tag{49}$$

Given the appropriate reference values and exponents, the above expressions provide the doppler shift and spread for each ionosphere return. However, since the reference values and exponents usually are not known, we must make some assumptions to obtain "default" values for use in the absence of more accurate data. It will be assumed that the doppler shift and spread are the same for the high ray and low ray and for both magnetoionic components of the E and F region returns. The following table gives the "default" reference values and exponents to be used for the E and F regions. These values are based upon measurements made by Watterson, et. al. [1].

Region	ν_r(Hz)	$2\sigma_r$(Hz)	f_r(mHz)	α	β
E	0.01	0.02	9.30	1.0	1.0
F	0.01	0.15	9.30	1.0	1.0

Table 1. Default Values for Doppler Shift and Doppler Spread

D. Computer Program

The Fortran program used to calculate the foregoing channel parameters is presented in Appendix B. The basic output of this program is a listing of the input parameters and a tabular presentation of the computed channel parameters. A typical output is shown in Table 2. The indexing convention used for the returns is presented in Table 3 and a glossary of the terms used in the output is presented in Table 4.

A modified version of this output can be produced in which all returns whose attenuation exceeds some threshold value above the minimum attenuation are omitted. Those modes for which there is no return are likewise omitted. Using this modified output one can identify those returns that are of primary interest for communication purposes. An example is shown in Table 5 using the same inputs that produced Table 2 and an attenuation threshold of 40 dB.

A synthetic ionogram also has been produced using the program. The resulting curves are presented in Figure 5 and include returns for one, two and three hops together with the corresponding attenuations.

Table 2. Typical Output Listing for Computed Channel Parameters (Includes All Returns for Six or Less Hops)

SUMMARY OF TRANSMISSION PARAMETERS

SIGNAL PARAMETERS

CARRIER FREQ. = 5.000 MHZ. BANDWIDTH = 100.00 KHZ.

TRANSMITTER/RECEIVER PARAMETERS

LOCATION OF TRANSMITTER: LONGITUDE = 150.00 DEGREES (+ FOR WEST, - FOR EAST)
LATITUDE = 30.00 DEGREES (+ FOR NORTH, - FOR SOUTH)
DISTANCE BETWEEN TRANSMITTER AND RECEIVER = 500.00 KM.
DIRECTION FROM TRANSMITTER TO RECEIVER = 0.00 DEGREES (MEASURED FROM TRUE NORTH)
TRANSMITTER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
RECEIVER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
SEA STATE = 1

IONOSPHERE PARAMETERS

SUNSPOT NUMBER = 100.00
SOLAR ZENITH ANGLE = 45.00 DEGREES
DAYTIME ELECTRON DENSITY PROFILE
E REGION: HEIGHT OF MAX. ELECTRON DENSITY = 110.00 KM.
SEMITHICKNESS OF PARABOLA = 20.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 2.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 2.630 MHZ.
F REGION: HEIGHT OF MAX. ELECTRON DENSITY = 250.00 KM.
SEMITHICKNESS OF PARABOLA = 50.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 8.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 8.537 MHZ.
NONDEVIATIVE ABSORPTION CONSTANTS
MULTIPLICATIVE CONSTANT = 215.00, SUNSPOT NUMBER MULTIPLIER = 0.003500, SOLAR ZENITH ANGLE EXPONENT = 0.750
DOPPLER REFERENCE VALUES
E REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.0000 SPREAD = 0.0200 HZ, SPREAD EXP. = 1.0000 REF. FREQ. = 9.300 MHZ.
F REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.0000 SPREAD = 0.1500 HZ, SPREAD EXP. = 1.0000 REF. FREQ. = 9.300 MHZ.

COMPUTED CHANNEL PARAMETERS

GROUNDWAVE: DELAY = 1.667E-03 SEC. ATTENUATION = 1.200E+02 DB.
IONOSPHERE RETURNS:

MODE	SOLUTION INDICATOR (0=NO SOLN)	RAY ANGLE (DEGREES)	PATH LENGTH (KM)	CARRIER DELAY (SEC)	CARRIER PHASE (CYCLES)	SIGNAL DELAY (SEC)	AMPLITUDE DISTORTION (SEC/MHZ)	PHASE DISTORTION (SEC/MHZ**2)	ATTENUATION (DB)	DOPPLER SHIFT (HZ)	DOPPLER SPREAD (HZ)
1EO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
1EO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
1FO LOW	1.0	46.91	684.68	2.282E-03	4.094E-01	2.096E-03	5.760E-05	1.346E-05	1.265E+02	5.376E-03	8.065E-02
1FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
1FX LOW	1.0	67.53	541.07	1.809E-03	7.755E-01	1.933E-03	1.098E-04	9.042E-05	1.619E+02	5.376E-03	1.075E-02
1FX HIGH	1.0	58.39	587.19	1.957E-03	4.458E-01	1.161E-03	7.306E-05	-1.595E-04	1.405E+02	5.376E-03	1.075E-02
1FX LOW	1.0	44.11	718.30	2.394E-03	6.671E-01	1.824E-03	1.434E-04	2.562E-05	1.401E+02	5.376E-03	8.065E-02
1FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2EO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2EO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2FO LOW	1.0	28.55	1046.08	3.487E-03	6.699E-01	3.583E-03	1.991E-04	4.136E-05	1.471E+02	1.075E-02	1.140E-01
2FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.

Table 2. Typical Output Listing for Computed Channel Parameters (Includes All Returns for Six or Less Hops). (Contd)

2EX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
2FX LOW	1.0	27.51	1082.65	3.609E-03	1.263E-01	3.196E-03	2.474E-04	5.244E-05	1.745E+02	1.075E-02	1.140E-01
2FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FO LOW	1.0	19.86	1471.76	4.906E-03	3.858E-01	5.305E-03	3.744E-04	7.501E-05	1.674E+02	1.613E-02	1.397E-01
3FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
3FX LOW	1.0	19.26	1515.47	5.052E-03	9.167E-01	4.737E-03	4.052E-04	8.120E-05	2.060E+02	1.613E-02	1.397E-01
3FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FO LOW	1.0	15.12	1916.91	6.390E-03	4.819E-01	7.068E-03	5.453E-04	1.080E-04	1.864E+02	2.151E-02	1.613E-01
4FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
4FX LOW	1.0	14.71	1959.42	6.565E-03	7.092E-01	6.310E-03	5.655E-04	1.101E-04	2.403E+02	2.151E-02	1.613E-01
4FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FO LOW	1.0	12.18	2369.95	7.900E-03	1.331E-01	8.839E-03	7.112E-04	1.401E-04	2.044E+02	2.688E-02	1.803E-01
5FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5EX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
5FX LOW	1.0	11.86	2432.10	8.107E-03	9.682E-01	7.890E-03	7.239E-04	1.389E-04	2.715E+02	2.688E-02	1.803E-01
5FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6EO LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FO LOW	1.0	10.19	2826.92	9.423E-03	2.850E-01	1.061E-02	8.736E-04	1.716E-04	2.217E+02	3.226E-02	1.975E-01
6FO HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FX LOW	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.
6FX LOW	1.0	9.93	2899.17	9.664E-03	4.629E-01	9.471E-03	8.806E-04	1.675E-04	3.020E+02	3.226E-02	1.975E-01
6FX HIGH	0.0	0.00	0.00	0.	0.	0.	0.	0.	0.	0.	0.

ALL IONOSPHERE RETURNS WHOSE ATTENUATION IS GREATER THAN 40.00 DB. ABOVE THE MINIMUM ATTENUATION ARE IGNORED

Table 3. Index Convention for Ionosphere Returns

Index	Number of Hops	Region	Ordinary or Extraordinary	Low Ray or High Ray
1	1	E	O	LOW
2	1	E	O	HIGH
3	1	F	O	LOW
4	1	F	O	HIGH
5	1	E	X	LOW
6	1	E	X	HIGH
7	1	F	X	LOW
8	1	F	X	HIGH
9	2	E	O	LOW
10	2	E	O	HIGH
11	2	F	O	LOW
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
44	6	F	O	HIGH
45	6	E	X	LOW
46	6	E	X	HIGH
47	6	F	X	LOW
48	6	F	X	HIGH

Table 4. Glossary of Terms Used for Computed Channel Parameters

Mode	Identifies particular ionosphere return (e.g., 2FO LOW is the two-hop, F region, ordinary wave, low ray return)
Solution Indicator	This equals 1 if there is a return for the particular mode and equals 0 if there is no return
Ray Angle	θ_j
Total Path Length	D_j
Carrier Delay	$\tau(f_c)$
Carrier Phase	The fractional part of $D_j^{(o)}$
Signal Delay	$D_j^{(1)}$
Amplitude Distortion	$D_j^{(2)}$
Phase Distortion	$D_j^{(3)}$
Attenuation	L_j
Doppler Shift	ν_j
Doppler Spread	$2\sigma_j$

Table 5. Typical Output Listing For Computed Channel Parameters

(Includes Only Those Returns Having Attenuations Within 40dB Of The Smallest Nonzero Attenuation)

SUMMARY OF TRANSMISSION PARAMETERS

SIGNAL PARAMETERS

CARRIER FREQ. = 5.000 MHZ. BANDWIDTH = 100.00 KHZ.

TRANSMITTER/RECEIVER PARAMETERS

LOCATION OF TRANSMITTER: LONGITUDE = 150.00 DEGREES (+ FOR WEST, - FOR EAST)
LATITUDE = 30.00 DEGREES (+ FOR NORTH, - FOR SOUTH)
DISTANCE BETWEEN TRANSMITTER AND RECEIVER = 500.00 KM.
DIRECTION FROM TRANSMITTER TO RECEIVER = 0.00 DEGREES (MEASURED FROM TRUE NORTH)
TRANSMITTER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
RECEIVER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)
SEA STATE = 1

IONOSPHERE PARAMETERS

SUNSPOT NUMBER = 100.00
SOLAR ZENITH ANGLE = 45.00 DEGREES
DAYTIME ELECTRON DENSITY PROFILE
E REGION: HEIGHT OF MAX. ELECTRON DENSITY = 110.00 KM.
SEMITHICKNESS OF PARABOLA = 20.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 2.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 2.630 MHZ.
F REGION: HEIGHT OF MAX. ELECTRON DENSITY = 250.00 KM.
SEMITHICKNESS OF PARABOLA = 50.00 KM.
PENETRATION FREQUENCY, ORDINARY WAVE = 8.000 MHZ.
PENETRATION FREQUENCY, EXTRAORDINARY WAVE = 8.537 MHZ.
NONDEVIATIVE ABSORPTION CONSTANTS
MULTIPLICATIVE CONSTANT = 215.00, SUNSPOT NUMBER MULTIPLIER = 0.003500, SOLAR ZENITH ANGLE EXPONENT = 0.750
DOPPLER REFERENCE VALUES
E REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.000; SPREAD = 0.0200 HZ, SPREAD EXP. = 1.000; REF. FREQ. = 9.300 MHZ.
F REGION: SHIFT = 0.0100 HZ, SHIFT EXP. = 1.000; SPREAD = 0.1500 HZ, SPREAD EXP. = 1.000; REF. FREQ. = 9.300 MHZ.

COMPUTED CHANNEL PARAMETERS

GROUNDWAVE: DELAY = 1.667E-03 SEC. ATTENUATION = 1.200E+02 DB.
IONOSPHERE RETURNS:

MODE	SOLUTION INDICATOR (0=NO SOLN)	RAY ANGLE (DEGREES)	PATH LENGTH (KM)	CARRIER DELAY (SEC)	CARRIER PHASE (CYCLES)	SIGNAL DELAY (SEC)	AMPLITUDE DISTORTION (SEC/MHZ)	PHASE DISTORTION (SEC/MHZ**2)	ATTENUATION (DB)	DOPPLER SHIFT (HZ)	DOPPLER SPREAD (HZ)
1FO LOW	NA	46.91	684.68	2.282E-03	4.094E-01	2.096E-03	5.760E-05	1.346E-05	1.265E+02	5.376E-03	8.065E-02
1FX LOW	NA	67.53	541.07	1.804E-03	7.755E-01	1.933E-03	1.098E-04	9.042E-05	1.619E+02	5.376E-03	1.075E-02
1EX HIGH	NA	58.38	587.19	1.957E-03	4.458E-01	1.161E-03	7.306E-05	-1.595E-04	1.465E+02	5.376E-03	1.075E-02
1FX LOW	NA	44.11	718.30	2.394E-03	6.671E-01	1.824E-03	1.434E-04	2.562E-05	1.401E+02	5.376E-03	8.065E-02
2FO LOW	NA	28.55	1046.08	3.487E-03	6.699E-01	3.583E-03	1.991E-04	4.136E-05	1.471E+02	1.075E-02	1.140E-01

ALL IONOSPHERE RETURNS WHOSE ATTENUATION IS GREATER THAN 40.00 DB. ABOVE THE MINIMUM ATTENUATION ARE IGNORED

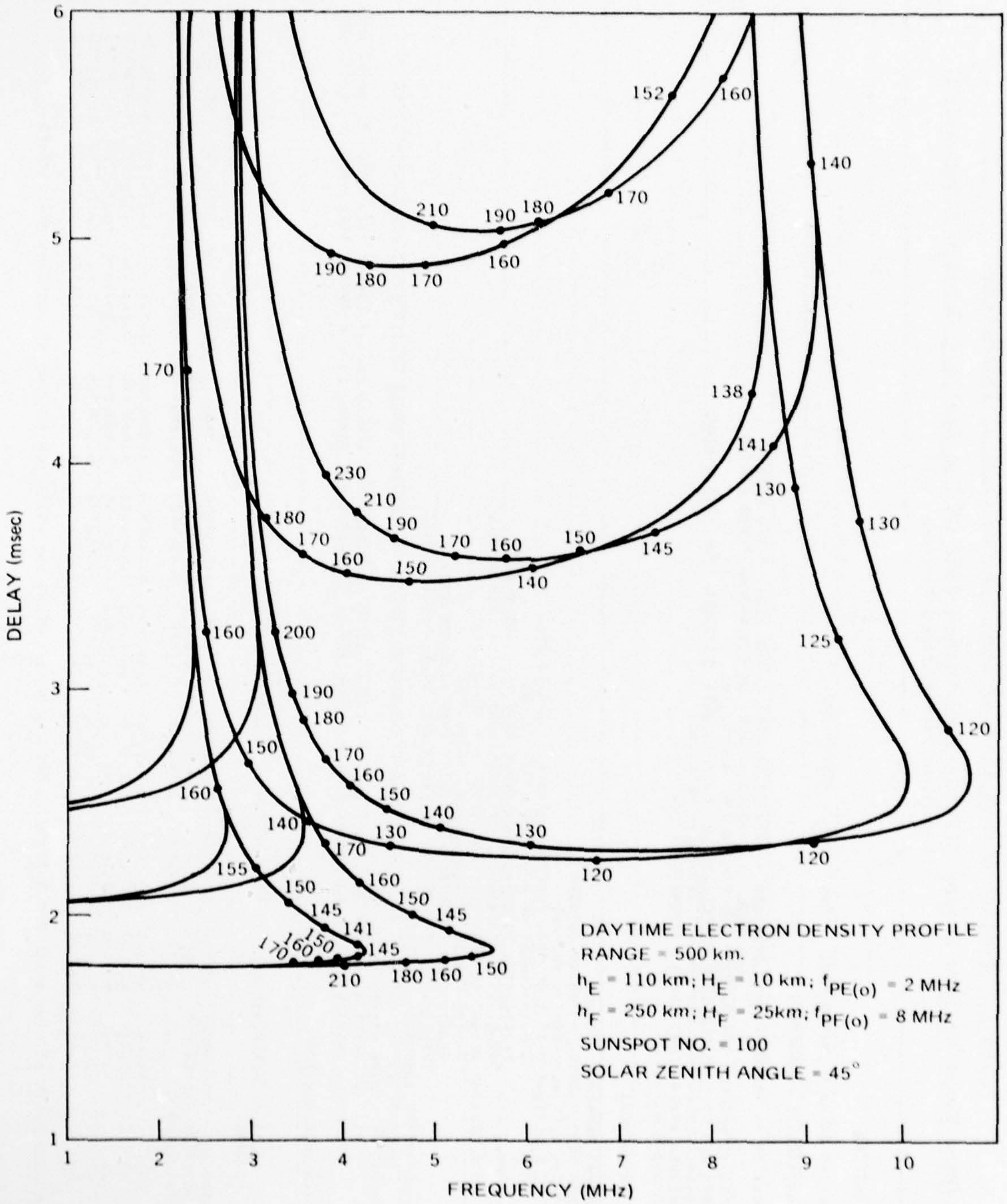

Figure 5. Computed Ionosphere Returns for One, Two and Three Hops
(Numbers on Curves are Attenuation in dB)

IV. SIGNAL ANALYSIS

This section is concerned with the signal analysis part of the simulator and the approximations used in its implementation. We will assume that the transmitted signal, s(t), is bandlimited with (two sided) bandwidth BW. The received signal, r(t), is not strictly bandlimited because it is multiplied by the random gains, which are not bandlimited. However, since the doppler spreads are small compared with the signal bandwidth, the received signal can also be assumed to be bandlimited but with a slightly larger bandwidth than the transmitted signal. The error introduced by this assumption is negligible as long as the assumed bandwidth of the received signal is large compared with the sum of the transmitted signal bandwidth and the largest doppler spread present. In the following we will assume that all signals have a bandwidth equal to twice that of the transmitted signal. This is sufficient to reduce aliasing errors to a negligible value. Denoting the sampling interval associated with this larger bandwidth by T_o we have

$$T_o = \frac{1}{2 \cdot BW} \quad . \tag{50}$$

As a consequence of the cardinal series representation for bandlimited functions, we require a knowledge of the signals involved only at the sample times nT_o, $n = 0, \pm 1, \cdots$. This greatly simplifies the computational and storage requirements. From (21) the received, sampled signal appears as

$$r(nT_o) = A_g \, s(nT_o - N_o T_o) + \sum_{j=1}^{N} \exp[-2\pi i D_j^{(o)}] \cdot G_j(nT_o) \, \hat{s}_j(nT_o - N_j T_o) \qquad n=0,1,\cdots \tag{51}$$

where the various delays have been replaced by the nearest integer multiple of T_o. That is, N_g is defined as the integer satisfying

$$D_g - \frac{1}{2} T_o \leq N_g T_o < D_g + \frac{1}{2} T_o \tag{52}$$

and the N_j's are defined as the integers satisfying

$$D_j^{(1)} - \frac{1}{2} T_o \leq N_j T_o < D_j^{(1)} + \frac{1}{2} T_o, \quad j=1, \cdots N. \tag{53}$$

It remains to determine the sampled, distorted signals $\hat{s}_j(nT_o)$ and the sampled random path gains $G_j(nT_o)$.

A. <u>Delay Distortion</u>

The transmitted signal has the representation

$$s(t) = \sum_{n=-\infty}^{\infty} s(nT_o) \frac{\sin 2\pi BW(t-nT_o)}{2\pi BW(t-nT_o)} \tag{54}$$

and the impulse response $h_j(t)$, which we also assume to be band-limited, has the representation

$$h_j(t) = \sum_{n=-\infty}^{\infty} h_j(nT_o) \frac{\sin 2\pi BW(t-nT_o)}{2\pi BW(t-nT_o)}, \quad j=1, \cdots N. \tag{55}$$

We obtain the distorted waveform, $\hat{s}_j(t)$, by convolving $s(t)$ and $h_j(t)$. But $\hat{s}_j(t)$ also has an expansion of the above type and

$$\hat{s}_j(t) = \sum_{n=-\infty}^{\infty} \hat{s}_j(nT_o) \frac{\sin 2\pi BW(t-nT_o)}{2\pi BW(t-nT_o)}, \quad j=1, \cdots N. \tag{56}$$

We are interested here in the sample values $\hat{s}_j(nT_o)$ and these are given by the convolution

$$\hat{s}_j(nT_o) = \sum_{m=-\infty}^{\infty} s(nT_o-mT_o) \hat{h}_j(mT_o), \quad j=1, \cdots, N. \tag{57}$$

where

$$\hat{h}_j(mT_o) = T_o h_j(mT_o) = T_o \int_{-BW}^{BW} H_j(f) \exp\,[2\pi i f m T_o]df. \qquad (58)$$

$$j=1,\cdots N$$

The transfer function $H_j(f)$ is given by (11), and it will be approximated with the first two terms of the series appearing in the exponent, that is

$$H_j(f) \simeq \exp[-2\pi i f^2 D_j^{(2)} - 2\pi i f^3 D_j^{(3)}],\ j-1,\cdots N. \qquad (59)$$

After the change of variable $\eta = f/BW$, we obtain from (58)

$$\hat{h}_j(mT_o) \approx \frac{1}{2}\int_{-1}^{1} \exp[-i\Delta_j^{(2)}\eta^2 - i\Delta_j^{(3)}\eta^3]\cdot\exp[im\pi\eta]d\,\eta \qquad (60)$$

$$j=1,\cdots N$$

with

$$\Delta_j^{(2)} = 2\pi\ (BW)^2\ D_j^{(2)} \qquad (61)$$

$$\Delta_j^{(3)} = 2\pi\ (BW)^3\ D_j^{(3)}\ . \qquad (62)$$

Closed form expressions for (60) are not known and further approximations are required for the numerical determination of $\hat{h}_j(mT_o)$. Let $I(x)$ represent the general integral defining the impulse responses, that is

$$I(x) = \frac{1}{2}\int_{-1}^{1} \exp[-i\Delta^{(2)}\eta^2 - i\Delta^{(3)}\eta^3]\cdot\exp[ix\eta]d\eta\ . \qquad (63)$$

Expanding $\exp[-i\Delta^{(3)}\eta^3]$ in a power series we have the representation

$$I(x) = \sum_{p=0}^{\infty} \frac{(-i\Delta^{(3)})^p}{p!} I_p(x) \qquad (64)$$

where we have defined

$$I_p(x) = \frac{1}{2}\int_{-1}^{1} \eta^{3p} \exp[-i\Delta^{(2)}\eta^2]\cdot\exp[ix\eta]d\eta \quad . \qquad (65)$$

The integral $I_o(x)$ is readily expressed in terms of Fresnel integrals [24]. For the higher order integrals the following recurrence relation has been derived

$$\begin{aligned} I_p(x) = {} & \frac{i}{4\Delta^{(2)}} \exp[-i\Delta^{(2)}] \left\{ \exp[ix] + (-1)^p \exp[-ix] \right\} \\ & + \frac{x}{2\Delta^{(2)}} I_{p-1}(x) - i\frac{(p-1)}{2\Delta^{(2)}} I_{p-2}(x), \; p\geq 1. \end{aligned} \qquad (66)$$

where we have adopted the convention that $I_{-1}(x)=0$.

The impulse responses for the ionosphere returns can now be evaluated with relative ease. Starting with $I_o(x)$ and using (66) we determine as many of the terms $I_p(x)$ as are needed to adequately approximate the infinite sum (64). The integral $I_o(x)$ is evaluated using the rational approximation presented in [25]. Care must be exercised in implementing this procedure, however. The error incurred by using the rational approximation for $I_o(x)$ is approximately 1.0×10^{-9}. For large arguments ($x>5$) nearly equal functions are subtracted in (66) and this reduces the number of significant figures in the resulting expression. For the higher order terms this reduction in the number of significant figures is important and the computed values may become inaccurate. It is evident from (64) that this effect will be more noticeable when $\Delta^{(3)}$ is large. Experience has shown that values of x as large as 20-25 can be tolerated in most applications before the errors become excessive.

B. Random Gains

The random gains, $G_j(t)$, in the ionosphere returns are complex valued, zero mean, Gaussian processes whose correlation function and power spectral density are given by (cf. Eqns. 19 and 20)

$$C_{G_j}(\tau) = A_j^{\,2} \exp[-2\pi^2\sigma_j^{\,2}\tau^2 + 2\pi i\nu_j\tau] \ , \ j=1,\cdots N \qquad (67)$$

and

$$P_{G_j}(f) = \frac{A_j^{\,2}}{\sqrt{2\pi}\sigma_j} \exp\left[-\frac{(f-\nu_j)^2}{2\sigma_j^{\,2}}\right] \ , \ j=1,\cdots N. \qquad (68)$$

We wish to generate a random sequence with the above correlation function and we wish to do this directly in the time domain and not rely on Fourier transform techniques. Because the doppler spreads, σ_j, are small relative to the signal bandwidths, the time domain approach requires a large memory. To avoid these large memory requirements we will replace the original process with an approximation that can be generated more easily.

The approximate random gain will be denoted by $\tilde{G}_j(t)$ and it will be chosen to have the power spectral density

$$P_{\tilde{G}_j}(f) = \frac{8A_j^{\,2}}{3\pi\rho\sigma_j} \frac{1}{\left[1+\left(\frac{f-\nu_j}{\rho\sigma_j}\right)^2\right]^3} \ , \ j=1,\cdots,N \qquad (69)$$

and corresponding correlation function

$$C_{\tilde{G}_j}(\tau) = A_j^{\,2}\left[1 + 2\pi\rho\sigma_j|\tau| + \frac{1}{3}\left(2\pi\rho\sigma_j|\tau|\right)^2\right] \cdot\exp\left[-2\pi\rho\sigma_j|\tau| + 2\pi i\nu_j\tau\right] \ , \ j=1,\cdots,N. \qquad (70)$$

The parameter ρ is chosen so that (70) agrees with (67) over a reasonable range of arguments. The value $\rho = 2.1$ has been determined to be a good choice and this is the value that will be used

throughout the following. Normalized plots of $C_{G_j}(\tau)$ and $C_{\tilde{G}_j}(\tau)$ are compared in Figure 6.

It is obvious from (69) that the approximate process $\tilde{G}_j(t)$ is not bandlimited. However, the error introduced by assuming that the process has bandwidth BW will be negligible since $(\sigma_j/BW) << 1$. Accordingly we will treat $\tilde{G}_j(t)$ as if it is bandlimited, in which case we need only evaluate it at the sample times nT_o. Thus, the original problem has been reduced to the generation of a complex valued, zero mean, Gaussian sequence with correlation function

$$C_{\tilde{G}_j}(nT_o) = A_j^2\left[1 + 2\pi\rho\sigma_j T_o|n| + \frac{1}{3}\left(2\pi\rho\sigma_j T_o|n|\right)^2\right] \cdot \exp\left[-2\pi\rho\sigma_j T_o|n| + 2\pi i\nu_j T_o n\right]. \qquad n=0,\pm 1,\ldots \quad j=1,\cdots N \tag{71}$$

The technique used for the computer generation of the approximate process is discussed in Appendix C. A plot of the normalized, empirical correlation function of the computer-generated process is also displayed in Figure 6. Agreement between the computer-generated correlation function and the approximate correlation function (71) is excellent for the range of arguments considered.

C. Computer Program

A listing of the Fortran program used to implement the signal analysis part of the simulator is presented in Appendix D. Channel parameter information is read into the program via data cards that have been produced by the first part of the simulator. The sampled input values are read from a tape, the program calculates the sampled output values according to Eqn. (51) and writes the output on another tape. As an alternate, printed output also can be obtained, but, because the quantity of output data is large, the usefulness of this mode of presentation is severely limited.

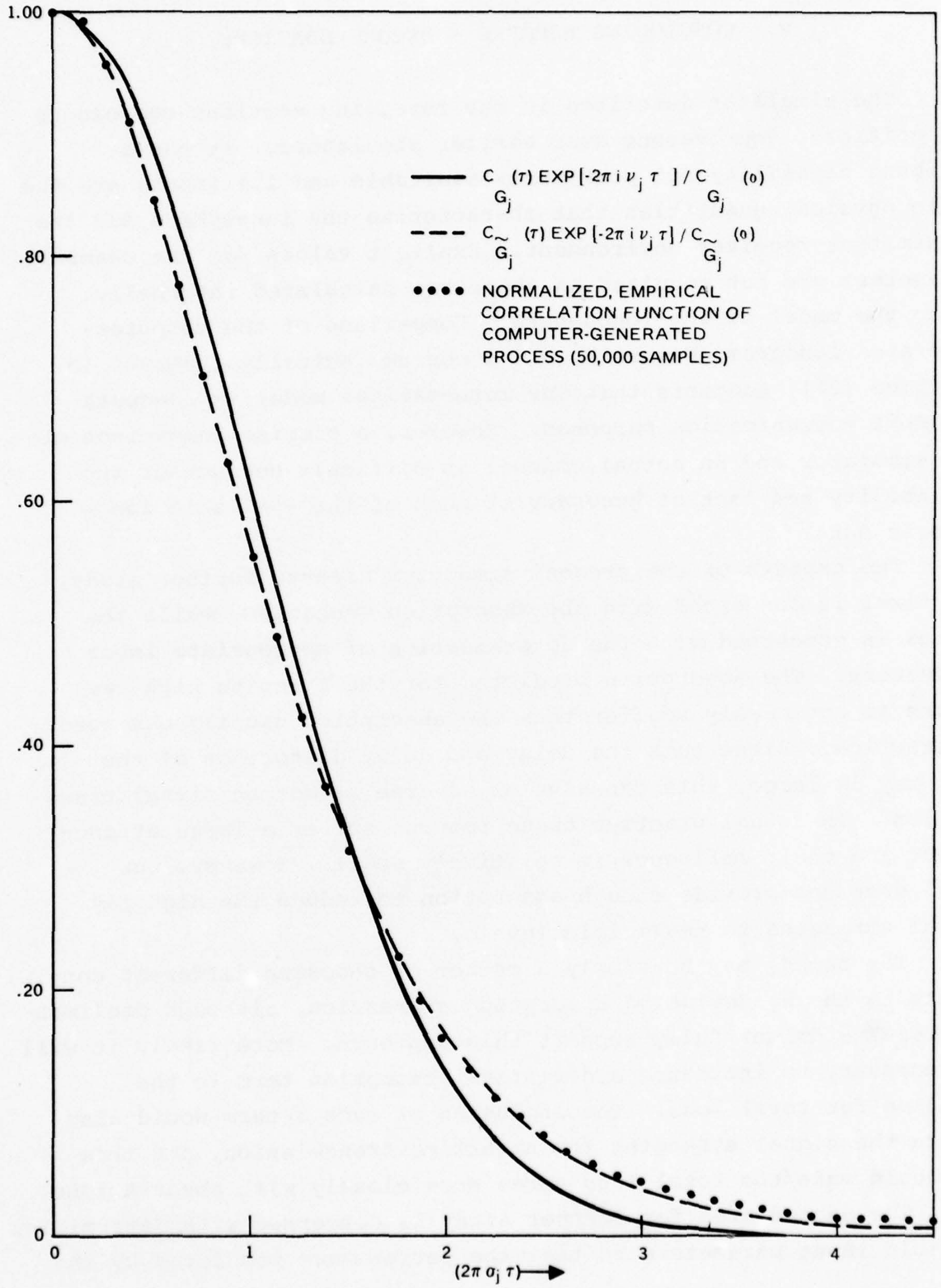

Figure 6. Normalized Correlation Function for the Random Gains

V. CONCLUDING REMARKS - RECOMMENDATIONS

The simulator described in the foregoing sections represents a significant improvement over earlier simulators. It has a wideband capability not previously available and its inputs are the basic physical quantities that characterize the ionosphere and the transmitter-receiver environment. Explicit values for the channel parameters are not required as these are calculated internally using the model of the ionosphere. Comparison of the computer-generated ionogram, Figure 5, and ionograms actually observed in practice [27], suggests that our mathematical model is adequate for most communication purposes. However, a precise comparison of the simulator and an actual channel is difficult because of the variability and lack of accuracy of much of the available ionospheric data.

Two aspects of the present simulator deserve further study. The first is concerned with the absorption mechanism, while the second is concerned with the determination of appropriate input parameters. The absorption predicted for the F region high ray return is noticeably smaller than the absorption usually observed in practice. Since both the delay and delay distortion of the high ray is large, this can have an adverse affect on signal transmission. In actual practice these returns suffer a large attenuation, and their influence is relatively small. However, our model does not provide enough absorption to reduce the high ray signal strengths to negligible levels.

The remedy may be simply a matter of choosing different constants in the nondeviative absorption expression, although preliminary results do not fully support this approach. More likely it will be necessary to introduce a deviative absorption term in the equation for total loss. The inclusion of such a term would also reduce the signal strengths for nighttime transmission, and this too would make the total loss agree more closely with observations.

The second area for further study is concerned with determining suitable input parameters so that the performance predicted by the

mathematical model agrees as well as possible with the actual observed performance. This involves matching actual and computer-generated observables such as doppler shift, doppler spread, attenuation and delay vs. frequency plots (ionograms) to determine the input parameters. Besides the obvious advantage of providing further justification of the assumptions used in the model, this parameter determination would provide the user with a set of inputs that correspond to a variety of actual environments.

The primary difficulty encountered in the determination of the input parameters is that an explicit functional relationship does not exist between the input parameters and the computed channel parameters; the channel parameters are determined by numerically solving a transcendental equation. As a consequence, techniques must be developed that are tailored to the ionosphere model. One approach is to select the most significant observable features and to determine the relationship between these features and the input parameters. For example, the nearly horizontal portions of an observed ionogram would be affected most by the heights of the regions and to a lesser degree by the thickness of the regions and their peak electron densities. On the other hand, the maximum operating frequency is likely to be more affected by the peak electron densities than the heights or thicknesses of the regions. A detailed study of these and other pertinent relationships would provide the user with the ability to determine input parameters corresponding to a variety of observed operating conditions. These same conditions could then be reproduced by the channel simulator as required for system studies.

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APPENDIX A
DELAY AND DELAY DERIVATIVES

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DELAY AND DELAY DERIVATIVES

In this appendix we present the equations for the delay and the first three delay derivatives. Recalling the definition of A(x) we have

$$A(x) = 2G\left(\frac{1-x^2}{x^2}\right)^{\frac{1}{2}} \tag{A.1}$$

where $x = \cos\theta$. We must solve the equation

$$R = A(x) \tag{A.2}$$

with R equal to the distance between transmitter and receiver. The solution then yields the delay via the equation

$$\tau = \frac{R}{c(1-x^2)^{\frac{1}{2}}} \tag{A.3}$$

with c the velocity of light. The delay derivatives are given by

$$\frac{d\tau}{df} = \frac{x\tau}{(1-x^2)} \frac{dx}{df} \tag{A.4}$$

$$\frac{d^2\tau}{df^2} = \frac{1+2x^2}{x^2\tau} \left(\frac{d\tau}{dx}\right)^2 + \frac{x\tau}{(1-x^2)} \frac{d^2x}{df^2} \tag{A.5}$$

and

$$\frac{d^3\tau}{df^3} = - \frac{3(1+x^2+2x^4)}{x^4\tau^2}\left(\frac{d\tau}{df}\right)^2 + \frac{3(1+2x^2)}{x^2\tau}\left(\frac{d\tau}{df}\right)\left(\frac{d^2\tau}{df^2}\right)$$

$$+ \frac{x\tau}{(1-x^2)} \frac{d^3x}{df^3} \quad . \tag{A.6}$$

Thus to determine the delay and delay derivatives we must solve (A.2) for x and then evaluate the first three derivatives of x with respect to f for use in (A.4) - (A.6). In the following these functions are listed for the two electron density profiles. The group height G(x) is obtained by evaluating the integral (24).

1. Nighttime

For the nighttime electron density the integral for the group height is evaluated and used in (A.1) to yield

$$A(x) = \begin{cases} 2\left(\dfrac{1-x^2}{x^2}\right)^{\frac{1}{2}} \left\{ h_E - 2H_E + H_E\, \hat{f}_E\, x \ln\left(\dfrac{1+\hat{f}_E x}{1-\hat{f}_E x}\right) \right\}, & \hat{f}_E x \leq 1 \\ 2\left(\dfrac{1-x^2}{x^2}\right)^{\frac{1}{2}} \left\{ h_F - 2H_F - 4H_E + 2H_E\, \hat{f}_E x \ln\left(\dfrac{\hat{f}_E x+1}{\hat{f}_E x-1}\right) \right. & \\ \qquad \left. + H_F\, \hat{f}_F x \ln\left(\dfrac{1+\hat{f}_F x}{1-\hat{f}_F x}\right) \right\}, & \hat{f}_E x > 1 \end{cases} \tag{A.7}$$

where

$$\hat{f}_E = \frac{f}{f_{pE}} \text{ and } \hat{f}_F = \frac{f}{f_{pF}} .$$

For $\hat{f}_E x \leq 1$ the derivatives of x with respect to frequency are:

$$\frac{dx}{df} = \left(\frac{x}{f}\right) \cdot \frac{R \cdot \left(\dfrac{x^2}{1-x^2}\right)^{\frac{1}{2}} - \left[2h_E - \dfrac{4H_E}{1-\hat{f}_E^{\,2} x^2}\right]}{R \cdot \left(\dfrac{x^2}{1-x^2}\right)^{\frac{3}{2}} + \left[2h_E - \dfrac{4H_E}{1-\hat{f}^{2}_E x^2}\right]} , \quad \hat{f}_E x \leq 1 \tag{A.8}$$

$$\frac{d^2x}{df^2} = \frac{1}{x}\left(\frac{dx}{df}\right)^2 - \frac{1}{f}\left(\frac{dx}{df}\right)$$

$$+\left(\frac{x}{f}\right)\cdot\frac{R\cdot\left(\frac{1}{1-x^2}\right)^{3/2}\left(\frac{dx}{df}\right)\left[1-\frac{3xf}{1-x^2}\,\frac{dx}{df}\right]+\frac{8H_E\; f\;\hat{f}^2{}_E\left(\frac{x}{f}+\frac{dx}{df}\right)}{\left(1-\hat{f}^2{}_E x^2\right)^2}}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2}+\left[2h_E-\frac{4H_E}{1-\hat{f}^2{}_E x^2}\right]}\;,\quad \hat{f}_E x \leq 1 \;. \tag{A.9}$$

$$\frac{d^3x}{df^3} = -\frac{1}{x^2}\left(\frac{dx}{df}\right)^3 + \frac{2}{x}\left(\frac{dx}{df}\right)\left(\frac{d^2x}{df^2}\right) + \frac{1}{f^2}\left(\frac{dx}{df}\right) - \frac{1}{f}\left(\frac{d^2x}{df^2}\right)$$

$$-\frac{1}{x}\left(\frac{x}{f}-\frac{dx}{df}\right)\left[\frac{d^2x}{df^2}-\frac{1}{x}\left(\frac{dx}{df}\right)^2+\frac{1}{f}\,\frac{dx}{df}\right]$$

$$-\frac{\left[\frac{d^2x}{df^2}-\frac{1}{x}\left(\frac{dx}{df}\right)^2+\frac{1}{f}\,\frac{dx}{df}\right]}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2}+\left[2h_E-\frac{4H_E}{1-\hat{f}_E{}^2x^2}\right]}\left\{\frac{3R\cdot x^2\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}}-\frac{8H_E x\hat{f}^2{}_E\left(\frac{x}{f}+\frac{dx}{df}\right)}{(1-\hat{f}^2{}_E x^2)^2}\right\}$$

$$+\frac{\left(\frac{x}{f}\right)}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2}+\left[2h_E-\frac{4H_E}{1-\hat{f}_E{}^2x^2}\right]}\left\{\frac{R}{(1-x^2)^{5/2}}\left[1-\frac{3xf}{1-x^2}\;\frac{dx}{df}\right]\right.$$

$$\cdot\left[3x\left(\frac{dx}{df}\right)^2+(1-x^2)\,\frac{d^2x}{df^2}\right]$$

$$-\frac{3Rf\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}}\left[\frac{1+x^2}{1-x^2}\left(\frac{dx}{df}\right)^2+\left(\frac{x}{f}\right)\frac{dx}{df}+x\,\frac{d^2x}{df^2}\right]$$

$$+\frac{32H_E\;x\,f\,\hat{f}_E{}^4\left(\frac{x}{f}+\frac{dx}{df}\right)^3}{(1-\hat{f}^2{}_E x^2)^3}$$

$$\left.+\frac{8H_E\;\hat{f}_E{}^2\left(\frac{x}{f}+\frac{dx}{df}\right)}{(1-\hat{f}^2{}_E x^2)^2}\left[\frac{x}{f}+5\,\frac{dx}{df}+2\,f\,\frac{d^2x}{df^2}\right]\right\}\;,\quad \hat{f}_E x \leq 1 \;. \tag{A.10}$$

For $\hat{f}_E x > 1$ these derivatives appear as

$$\frac{dx}{df} = \left(\frac{x}{f}\right) \cdot \frac{R \cdot \left(\frac{x^2}{1-x^2}\right)^{1/2} - \left[2h_F + \frac{8H_E}{\hat{f}^2{}_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2{}_F x^2}\right]}{R \cdot \left(\frac{x^2}{1-x^2}\right)^{3/2} + \left[2h_F + \frac{8H_E}{\hat{f}^2{}_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2{}_F x^2}\right]}, \quad \hat{f}_E x > 1 \qquad \text{(A.11)}$$

$$\frac{d^2x}{df^2} = \frac{1}{x}\left(\frac{dx}{df}\right)^2 - \frac{1}{f}\left(\frac{dx}{df}\right)$$

$$+\left(\frac{x}{f}\right) \cdot \frac{\frac{R \cdot \left(\frac{dx}{df}\right)}{(1-x^2)^{3/2}}\left[1 - \frac{3xf}{1-x^2}\frac{dx}{df}\right] + \frac{16H_E\, f\, \hat{f}^2{}_E \left(\frac{x}{f} + \frac{dx}{df}\right)^2}{\left(\hat{f}^2{}_E x^2 - 1\right)^2} + \frac{8H_F\, f\, \hat{f}^2{}_F \left(\frac{x}{f} + \frac{dx}{df}\right)^2}{\left(1-\hat{f}^2{}_F x^2\right)^2}}{R \cdot \left(\frac{x^2}{1-x^2}\right)^{3/2} + \left[2h_F + \frac{8H_E}{\hat{f}^2{}_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2{}_F x^2}\right]}, \quad \hat{f}_E x > 1 \qquad \text{(A.12)}$$

$$\frac{d^3x}{df^3} = -\frac{1}{x^2}\left(\frac{dx}{df}\right)^3 + \frac{2}{x}\left(\frac{dx}{df}\right)\left(\frac{d^2x}{df^2}\right) + \frac{1}{f^2}\left(\frac{dx}{df}\right) - \frac{1}{f}\left(\frac{d^2x}{df^2}\right)$$

$$- \frac{1}{x}\left(\frac{x}{f} - \frac{dx}{df}\right)\left[\frac{d^2x}{df^2} - \frac{1}{x}\left(\frac{dx}{df}\right)^2 + \frac{1}{f}\frac{dx}{df}\right]$$

$$- \frac{\left[\frac{d^2x}{df^2} - \frac{1}{x}\left(\frac{dx}{df}\right)^2 + \frac{1}{f}\frac{dx}{df}\right]}{R \cdot \left(\frac{x^2}{1-x^2}\right)^{3/2} + \left[2h_F + \frac{8H_E}{\hat{f}^2{}_E x^2 - 1} - \frac{4H_F}{1-\hat{f}^2{}_F x^2}\right]} \left\{ \frac{3Rx^2\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}} \right.$$

$$\left. - \frac{16H_E x\, \hat{f}^2{}_E \left(\frac{x}{f} + \frac{dx}{df}\right)}{\left(\hat{f}^2{}_E x^2 - 1\right)^2} - \frac{8H_F x\, \hat{f}^2{}_F \left(\frac{x}{f} + \frac{dx}{df}\right)}{\left(1-\hat{f}^2{}_F x^2\right)^2} \right\}$$

(cont.)

$$+\frac{\frac{x}{f}}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2}+\left[2h_F+\frac{8H_E}{\hat{f}^2{}_E x^2-1}-\frac{4H_F}{1-\hat{f}^2{}_F x^2}\right]}$$

$$\cdot\left\{\frac{R\cdot\left[1-\frac{3xf}{1-x^2}\frac{dx}{df}\right]}{(1-x^2)^{5/2}}\left[3x\left(\frac{dx}{df}\right)^2+(1-x^2)\left(\frac{d^2x}{df^2}\right)\right]\right.$$

$$-\frac{3Rf\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}}\left[\frac{1+x^2}{1-x^2}\left(\frac{dx}{df}\right)^2+\left(\frac{x}{f}\right)\frac{dx}{df}+x\frac{d^2x}{df^2}\right]$$

$$-\frac{64H_E\,x\,f\,\hat{f}^4{}_E\left(\frac{x}{f}+\frac{dx}{df}\right)^3}{(\hat{f}^2{}_E x^2-1)^3}+\frac{32H_F\,x\,f\,\hat{f}^4{}_F\left(\frac{x}{f}+\frac{dx}{df}\right)^3}{(1-\hat{f}^2{}_F x^2)^3}$$

$$+\frac{16H_E\,\hat{f}^2{}_E\left(\frac{x}{f}+\frac{dx}{df}\right)\left[\frac{x}{f}+5\frac{dx}{df}+2f\frac{d^2x}{df^2}\right]}{(\hat{f}^2{}_E x^2-1)^2}$$

$$+\frac{8H_F\,\hat{f}^2{}_F\left(\frac{x}{f}+\frac{dx}{df}\right)\left[\frac{x}{f}+5\frac{dx}{df}+2f\frac{d^2x}{df^2}\right]}{(1-\hat{f}^2{}_F x^2)^2},\quad f_E x>1. \qquad \text{(A.13)}$$

2. Daytime

The daytime electron density profile differs from the nighttime profile only for heights greater than h_E. Thus the daytime equations for $\hat{f}_E \le 1$ are the same as the nighttime equations. In this case

$$
A(x) = \begin{cases} 2\left(\dfrac{1-x^2}{x^2}\right)^{\frac{1}{2}} \left\{ h_E - 2H_E + H_E \hat{f}_E x \ln\left(\dfrac{1+\hat{f}_E x}{1-\hat{f}_E x}\right)\right\}, & \hat{f}_E x \leq 1 \\[2ex] 2\left(\dfrac{1-x^2}{x^2}\right)^{\frac{1}{2}} \left\{ h_E - 2H_E + H_E \hat{f}_E x \ln\left(\dfrac{\hat{f}_E x+1}{\hat{f}_E x-1}\right) \right. & \\[2ex] \qquad + \left[h_F - h_E - 2H_F (1-r^2)^{\frac{1}{2}}\right]\left(\dfrac{\hat{f}_E{}^2 x^2}{\hat{f}^2{}_E x^2 - 1}\right)^{\frac{1}{2}} & \\[2ex] \qquad \left. + \; 2H_F \hat{f}_F x \ln\left[\dfrac{(1-r^2)^{\frac{1}{2}} + (\hat{f}^2{}_F x^2 - r^2)^{\frac{1}{2}}}{(1-\hat{f}^2{}_F x^2)^{\frac{1}{2}}}\right]\right\}, & \hat{f}_E x > 1 \end{cases} \tag{A.14}
$$

where

$$
r^2 = \frac{f^2{}_{pE}}{f^2{}_{pF}} \quad .
$$

For $\hat{f}_E x \leq 1$ the derivatives of x with respect to frequency are given by (A.8) - (A.10). For $\hat{f}_E x > 1$ we have

$$
\frac{dx}{df} = \left(\frac{x}{f}\right) \frac{R \cdot \left(\dfrac{x^2}{1-x^2}\right)^{\frac{1}{2}} - B(x)}{R \cdot \left(\dfrac{x^2}{1-x^2}\right)^{\frac{3}{2}} + B(x)} \quad , \quad \hat{f}_E x > 1 \tag{A.15}
$$

with B(x) defined by

$$
B(x) = 2h_E + \frac{4H_E}{\hat{f}^2{}_E x^2 - 1} + 2\left(h_F - h_E - 2H_F\sqrt{1-r^2}\right) \cdot \left(\frac{\hat{f}^2{}_E x^2}{\hat{f}^2{}_E x^2 - 1}\right)^{\frac{3}{2}}
$$

$$
- \frac{4H_F \hat{f}^3{}_F x^3}{\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2 - r^2)^{\frac{1}{2}} + (\hat{f}^2{}_F x^2 - r^2)} - \frac{4H_F \hat{f}^3{}_F x^3}{1-\hat{f}^2{}_F x^2} \tag{A.16}
$$

$$\frac{d^2x}{df^2} = \frac{1}{x}\left(\frac{dx}{df}\right)^2 - \frac{1}{f}\frac{dx}{df}$$

$$+ \frac{\dfrac{R\left(\frac{x}{f}\right)\frac{dx}{df}}{(1-x^2)^{3/2}}\left[1-\dfrac{3xf}{1-x^2}\dfrac{dx}{df}\right] - \left(\dfrac{x}{f}+\dfrac{dx}{df}\right)\dfrac{dB(x)}{df}}{R\cdot\left(\dfrac{x^2}{1-x^2}\right)^{3/2} + B(x)}\ , \quad \hat{f}_E x > 1 \qquad \text{(A.17)}$$

$$\frac{dB(x)}{df} = \left(\frac{x}{f} + \frac{dx}{df}\right)\cdot\left\{\frac{8H_E\,\hat{f}^2{}_E\,x}{(\hat{f}^2{}_E x^2-1)^2} - 6\left(h_F - h_E - 2H_F\sqrt{1-r^2}\right.\ \frac{\hat{f}^3{}_E x^2}{(\hat{f}^2{}_E x^2-1)^{5/2}}\right.$$

$$- \frac{12H_F\,\hat{f}^3{}_F\,x^2}{\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2-r^2)^{1/2} + (\hat{f}^2{}_F x^2-r^2)}$$

$$+ \frac{4H_F\,\hat{f}^5{}_F\,x^4\left[2+\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2-r^2)^{-1/2}\right]}{\left[\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2-r^2)^{1/2} + (\hat{f}^2{}_F x^2-r^2)\right]^2}$$

$$\left.- \frac{4H_F\,\hat{f}^3{}_F\,x^2(3-\hat{f}^2{}_F x^2)}{(1-\hat{f}^2{}_F x^2)^2}\right\} \qquad \text{(A.18)}$$

$$\frac{d^3x}{df^3} = -\frac{1}{x^2}\left(\frac{dx}{df}\right)^3 + \frac{2}{x}\left(\frac{dx}{df}\right)\left(\frac{d^2x}{df^2}\right) + \frac{1}{f^2}\,\frac{dx}{df} - \frac{1}{f}\,\frac{d^2x}{df^2}$$

$$+ \frac{1}{R\cdot\left(\frac{x^2}{1-x^2}\right)^{3/2} + B(x)} \left\{- \frac{3R\,x\left(\frac{dx}{df}\right)}{(1-x^2)^{5/2}}\left[\frac{1+4x^2}{1-x^2}\left(\frac{dx}{df}\right)^2 + \left(\frac{x}{f}\right)\left(\frac{dx}{df}\right) + 3x\,\frac{d^2x}{df^2}\right]\right.$$

$$+ \frac{R}{f\cdot(1-x^2)^{3/2}}\left[\frac{1+2x^2}{1-x^2}\left(\frac{dx}{df}\right)^2 - \left(\frac{x}{f}\right)\frac{dx}{df} + x\,\frac{d^2x}{df^2}\right]$$

$$- \frac{1}{f}\,\frac{dB(x)}{df}\left[2\,\frac{dx}{df} - \frac{x}{f} + 2f\,\frac{d^2x}{df^2} - \frac{f}{x}\left(\frac{dx}{df}\right)^2\right]$$

$$\left. - \left(\frac{x}{f} + \frac{dx}{df}\right)\frac{d^2B(x)}{df^2}\right\}, \quad \hat{f}_E x>1 \qquad \text{(A.19)}$$

$$\frac{d^2B(x)}{df^2} = \frac{1}{f}\;\frac{\frac{dB(x)}{df}}{\left(\frac{x}{f} + \frac{dx}{df}\right)}\left[\frac{dx}{df} - \frac{x}{f} + f\,\frac{d^2x}{df^2}\right]$$

$$+ \left(\frac{x}{f} + \frac{dx}{df}\right)\left\{- \frac{8H_E\hat{f}^2{}_E}{(\hat{f}^2{}_E x^2-1)^3}\left[\hat{f}^2{}_E x^2\left(2\,\frac{x}{f} + 3\,\frac{dx}{df}\right) + 2\left(\frac{x}{f} + \frac{dx}{df}\right)\right]\right.$$

$$+ \frac{6\hat{f}^3{}_E\,x\left(h_F-h_E-2H_F\sqrt{1-r^2}\right)}{(\hat{f}^2{}_E x^2-1)^{7/2}}\left[\hat{f}^2{}_E x^2\left(2\,\frac{x}{f} + 3\,\frac{dx}{df}\right) + \left(3\,\frac{x}{f} + 2\,\frac{dx}{df}\right)\right]$$

(cont.)

$$+ \frac{12H_F \hat{f}^3{}_F x \left[-\left(3\frac{x}{f} + 2\frac{dx}{df}\right) + \hat{f}^2{}_F x^2 \left(\frac{x}{f} + \frac{dx}{df}\right) \left[2 + \sqrt{1-r^2}\left(\hat{f}^2{}_F x^2 - r^2\right)^{-\frac{1}{2}}\right]\right]}{\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2 - r^2)^{\frac{1}{2}} + (\hat{f}^2{}_F x^2 - r^2)}$$

$$+ \frac{4H_F \hat{f}^5{}_F x^3 \left(5\frac{x}{f} + 4\frac{dx}{df}\right) \left[2 + \sqrt{1-r^2}\left(\hat{f}^2{}_F x^2 - r^2\right)^{-\frac{1}{2}}\right]}{\left[\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2 - r^2)^{\frac{1}{2}} + (\hat{f}^2{}_F x^2 - r^2)\right]^2}$$

$$- \frac{4H_F \hat{f}^7{}_F x^5 \sqrt{1-r^2}\left(\frac{x}{f} + \frac{dx}{df}\right) \cdot \left(\hat{f}^2{}_F x^2 - r^2\right)^{-\frac{3}{2}}}{\left[\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2 - r^2)^{\frac{1}{2}} + (\hat{f}^2{}_F x^2 - r^2)\right]^2}$$

$$- \frac{8H_F \hat{f}^7{}_F x^5 \left(\frac{x}{f} + \frac{dx}{df}\right) \left[2 + \sqrt{1-r^2}\left(\hat{f}^2{}_F x^2 - r^2\right)^{-\frac{1}{2}}\right]^2}{\left[\sqrt{1-r^2}\,(\hat{f}^2{}_F x^2 - r^2)^{\frac{1}{2}} + (\hat{f}^2{}_F x^2 - r^2)\right]^3}$$

$$- \frac{16H_F \hat{f}^5{}_F x^3 \left(\frac{x}{f} + \frac{dx}{df}\right)(3 - \hat{f}^2{}_F x^2)}{(1 - \hat{f}^2{}_F x^2)^3}$$

$$- \frac{4H_F \hat{f}^3{}_F x \left[3\left(3\frac{x}{f} + 2\frac{dx}{df}\right) - \hat{f}^2{}_F x^2 \left(5\frac{x}{f} + 4\frac{dx}{df}\right)\right]}{(1 - \hat{f}^2{}_F x^2)^2} \Bigg\} \qquad \text{(A.20)}$$

APPENDIX B

PROGRAM FOR CALCULATING CHANNEL PARAMETERS

```
B6700/B7700   F O R T R A N   C O M P I L A T I O N   M A R K   2.8.000    MONDAY, 09/26/77    03:00 PM
                                                                   START OF SEGMENT 002
                                                               FORMAT SEGMENT IS 000C LONG
                                                               FORMAT SEGMENT IS 000D LONG
                                                               FORMAT SEGMENT IS 0005 LONG
      DIMENSION T(48,14),P(4,8),A(3)                            L  002:000010
      COMMON HE,HEM,HF,HFM,TIME                                 L  002:000010
                                                                L  002:000010
C        THIS PROGRAM COMPUTES CHANNEL PARAMETERS FROM BASIC INPUTS   L  002:000010
                                                                L  002:000010
C        GLOSSARY OF MOST IMPORTANT VARIABLES                   L  002:000010
C              FC     = CARRIER FREQUENCY (MHZ.)                L  002:000010
C              BW     = BANDWIDTH (KHZ.)                        L  002:000010
C              XLONG  = LONGITUDE OF TRANSMITTER (DEGREES) + FOR WEST,   L  002:000010
C                       - FOR EAST)                             L  002:000010
C              XLAT   = LATITUDE OF TRANSMITTER (DEGREES) + FOR NORTH,   L  002:000010
C                       - FOR SOUTH)                            L  002:000010
C              R      = DISTANCE BETWEEN TRANSMITTER AND RECEIVER (KM.)   L  002:000010
C              RAG    = DIRECTION FROM TRANSMITTER TO RECEIVER (DEGREES)  L  002:000010
C                       MEASURED IN GEOGRAPHIC COORDINATES)     L  002:000010
C              RAM    = DIRECTION FROM TRANSMITTER TO RECEIVER (DEGREES)  L  002:000010
C                       MEASURED IN MAGNETIC COORDINATES)       L  002:000010
C              ISEA   = SEA STATE                               L  002:000010
C              SPT    = SUNSPOT NUMBER                          L  002:000010
C              ZEN    = SOLAR ZENITH ANGLE (DEGREES)            L  002:000010
C              TIME   = TIME OF DAY ('DAY' OR 'NITE')           L  002:000010
C              HEM    = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)   L  002:000010
C              4HE    = THICKNESS OF E REGION (KM.)             L  002:000010
C              HFM    = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.)   L  002:000010
C              4HF    = THICKNESS OF F REGION (KM.)             L  002:000010
C              FPEO   = PENETRATION FREQ. OF E REGION O-WAVE (MHZ.)   L  002:000010
C              FPFO   = PENETRATION FREQ. OF F REGION O-WAVE (MHZ.)   L  002:000010
C              FPEX   = PENETRATION FREQ. OF E REGION X-WAVE (MHZ.)   L  002:000010
C              FPFX   = PENETRATION FREQ. OF F REGION X-WAVE (MHZ.)   L  002:000010
C              ABSMLT= MULTIPLICATIVE CONST. FOR NONDEVIATIVE ABSORPTION   L  002:000010
C              SPTMLT= SUNSPOT NO. MULTIPLIER FOR NONDEVIATIVE ABSORPTION  L  002:000010
C              SOLEXP= SOLAR ZEN. ANG. EXPNT. FOR NONDEVIATIVE ABSORPTION  L  002:000010
C              SHIFTE= E REGION REFERENCE DOPPLER SHIFT (HZ.)   L  002:000010
C              EXSHTE= E REGION DOPPLER SHIFT EXPONENT          L  002:000010
C              SPRDE = E REGION REFERENCE DOPPLER SPREAD (HZ.)  L  002:000010
C              EXSPRE= E REGION DOPPLER SPREAD EXPONENT         L  002:000010
C              REFRQE= E REGION DOPPLER REFERENCE FREQ. (MHZ.)  L  002:000010
C              SHIFTF= F REGION REFERENCE DOPPLER SHIFT (HZ.)   L  002:000010
C              EXSHTF= F REGION DOPPLER SHIFT EXPONENT          L  002:000010
C              SPRDF = F REGION REFERENCE DOPPLER SPREAD (HZ.)  L  002:000010
C              EXSPRF= F REGION DOPPLER SPREAD EXPONENT         L  002:000010
C              REFRQF= F REGION DOPPLER REFERENCE FREQ. (MHZ.)  L  002:000010
C              ATT    = ATTENUATION (DB.)                       L  002:000010
C              ATTDIF= ATTENUATION DIFFERENCE BETWEEN STRONGEST RETURN   L  002:000010
C                       FROM IONOSPHERE AND WEAKEST RETURN RETAINED      L  002:000010
C              NPATH = NO. OF IONOSPHERE RETURNS RETAINED       L  002:000010
C              GDELAY= GROUNDWAVE DELAY (SEC.)                  L  002:000010
C              GATTEN= GROUNDWAVE ATTENUATION (DB)              L  002:000010
C              TPRINT= PRINT SELECTOR FOR T(I,J) ARRAY ('ALL' OR 'RET')   L  002:000010
C              NPUNCH= PUNCH SELECTOR. PUNCHES DATA CARDS FOR USE IN     L  002:000010
C                       SIGNAL ANALYSIS PART OF SIMULATION IF NPUNCH=1,  L  002:000010
C                       DOES NOT PUNCH DATA CARDS IF NPUNCH=0.  L  002:000010
                                                                L  002:000010
C        ARRAY T(I,J) CONTAINS THE FOLLOWING PARAMETERS         L  002:000010
```

```
C              I = N: INDEX NUMBER FOR PATH                                C  002:000010
C              J = 1: NUMBER OF HOPS                                       C  002:000010
C                = 2: REGION AND MAGNETOIONIC INDICATOR (EO = E REGION -   C  002:000010
C                     ORDINARY WAVE) SIMILARLY FOR EX, FO, AND FX)         C  002:000010
C                = 3: 'HIGH' (FOR HIGH RAY) OR 'LOW' (FOR LOW RAY)         C  002:000010
C                = 4: SOLUTION INDICATOR (0=NO SOLUTION, 1=SOLUTION)       C  002:000010
C                = 5: THETA = RAY ANGLE MEASURED FROM VERTICAL (DEGREES)   C  002:000010
C                = 6: PATH LENGTH (KM.)                                    C  002:000010
C                = 7: DELAY AT CARRIER FREQUENCY (SEC.)                    C  002:000010
C                = 8: RECEIVED CARRIER PHASE (CYCLES)                      C  002:000010
C                = 9: SIGNAL DELAY (SEC.)                                  C  002:000010
C                =10: AMPLITUDE DISTORTION (SEC/MHZ)                       C  002:000010
C                =11: PHASE DISTORTION (SEC/MHZ**2)                        C  002:000010
C                =12: ATTENUATION (DB.)                                    C  002:000010
C                =13: DOPPLER SHIFT (HZ.)                                  C  002:000010
C                =14: DOPPLER SPREAD (HZ.)                                 C  002:000010
                                                                           C  002:000010
      PI=3.14159265359                                                     C  002:000010
      TN='NITE'                                                            C  002:000510
      PREF='RET'                                                           C  002:000713
C          READ DATA                                                       C  002:000913
      READ/,FC,BW                                                          C  002:000913
                                                                              FIB IS 0006 LONG
      READ/,XLONG,XLAT                                                     C  002:001310
      READ/,R                                                              C  002:001010
      READ/,RAG                                                            C  002:002510
      READ/,ISEA                                                           C  002:002010
      READ/,SPT                                                            C  002:003510
      READ/,ZEN                                                            C  002:003010
      READ(5,909)TIME                                                      C  002:004510
      READ/,HEM,HE,FPEO                                                    C  002:004012
      READ/,HFM,HF,FPFO                                                    C  002:005A10
      READ/,ABSMLT,SPTMLT,SOLEXP                                           C  002:006810
      READ/,SHIFTE,EXSHTE,SPRDE,EXSPRE,RFRFQE                              C  002:007410
      READ/,SHIFTF,EXSHTF,SPRDF,EXSPRF,RFRFQF                              C  002:008410
      READ/,ATTDIF                                                         C  002:009410
      READ/,GDELAY,GATTEN                                                  C  002:009C10
      READ(5,909)TPRINT                                                    C  002:00A610
      READ/,NPUNCH                                                         C  002:00AD12
C          END READ DATA                                                   C  002:00B510
C          COMPUTE PENETRATION FREQUENCIES FOR EXTRAORDINARY WAVES         C  002:00B510
      CALL GETONA(XLAT,XLONG,XLATM,XLONGM)                                 C  002:00B510
      SINPHI=SIN((PI/180.)*XLATM)                                          C  002:00B714
      XMLAT=.87*SQRT(1.+3.*(SINPHI**2))                                    C  002:00BA11
      FHEX=XMLAT*((6370./(6370.+HEM))**3)                                  C  002:00BE15
      FHFX=XMLAT*((6370./(6370.+HFM))**3)                                  C  002:00C210
      FPEX=.5*FHEX+SQRT((FPEO**2)+(.25*(FHEX**2)))                         C  002:00C514
      FPFX=.5*FHFX+SQRT((FPFO**2)+(.25*(FHFX**2)))                         C  002:00C815
C          END COMPUTATION OF PENETRATION FREQS. FOR EXTRAORDINARY WAVES   C  002:00D115
C          COMPUTE BEARING OF RECEIVER IN MAGNETIC COORDINATES             C  002:00D115
      CALL BNGMAG(XLAT,XLONG,R,RAG,RAM)                                    C  002:00D115
C          END COMPUTATION OF RECEIVER BEARING                             C  002:00D510
      WRITE(6,910)                                                         C  002:00D510
                                                                              FIB IS 0006 LONG
      WRITE(6,911)                                                         C  002:00D912
      WRITE(6,912)FC,BW                                                    C  002:00DD12
      WRITE(6,913)                                                         C  002:00E512
      WRITE(6,915)XLONG                                                    C  002:00E912
      WRITE(6,916)XLAT                                                     C  002:00F012
      WRITE(6,917)R                                                        C  002:00F712
```

```
      WRITE(6,918)MAG                                            C  002:00FL:2
      WRITE(6,919)                                               C  002:0105:2
      WRITE(6,920)                                               C  002:010V:2
      WRITE(6,921)ISEA                                           C  002:010U:2
      WRITE(6,922)                                               C  002:0114:2
      WRITE(6,923)SPT                                            C  002:0118:2
      IF(TIME.IS.TN)GOTO9                                        C  002:011F:2
      WRITE(6,932)/EN                                            C  002:0121:0
      GOTO10                                                     C  002:0127:2
 9    WRITE(6,933)                                               C  002:0127:5
 10   IF(TIME.IS.TN)GOTO11                                       C  002:012C:2
      WRITE(6,924)                                               C  002:012E:0
      GOTO12                                                     C  002:0132:2
 11   WRITE(6,925)                                               C  002:0132:5
 12   WRITE(6,926)HEM                                            C  002:0137:2
      SE=2.*HE                                                   C  002:013E:2
      WRITE(6,927)SE                                             C  002:013F:5
      WRITE(6,928)FPED                                           C  002:0146:2
      WRITE(6,930)FPEX                                           C  002:014D:2
      WRITE(6,929)HFM                                            C  002:0154:2
      SF=2.*HP                                                   C  002:015B:2
      WRITE(6,927)SF                                             C  002:015D:0
      WRITE(6,928)FPFO                                           C  002:0163:2
      WRITE(6,930)FPFX                                           C  002:016A:2
      WRITE(6,953)                                               C  002:0171:2
      WRITE(6,954)ABSMIT,SPTMLT,SOLEXP                           C  002:0175:2
      WRITE(6,945)                                               C  002:017F:2
      WRITE(6,946)SHIFTE,FXSHTE,SPRDE,EXSPRE,HFREQE              C  002:0183:2
      WRITE(6,947)SHIFTF,FXSHTF,SPRDF,EXSPRF,HFREQF              C  002:0190:2
      WRITE(6,900)                                               C  002:0190:2
      WRITE(6,934)                                               C  002:01A1:2
      WRITE(6,900)                                               C  002:01A5:2
      WRITE(6,935)GDELAY,GAFTEN                                  C  002:01A9:2
      WRITE(6,936)                                               C  002:01B1:2
C           DO 60 COMPUTES THE ARRAY T(I,J)                      C  002:01B5:2
      DO60NHOP=1,6                                               C  002:01B5:2
      XNHOP=FLOAT(NHOP)                                          C  002:01B6:0
      RH=R/XNHOP                                                 C  002:01B6:5
      NNHOP=8*(NHOP-1)                                           C  002:01B8:1
      DO35I=1,8                                                  C  002:01BA:0
      II=I+NNHOP                                                 C  002:01B8:0
      T(II,1)=NHOP                                               C  002:01BC:3
 35   CONTINUE                                                   C  002:01BD:5
      I=I+NNHOP                                                  C  002:01C0:0
C           ASSIGN LABELS TO EACH TRANSMISSION MODE              C  002:01C1:2
      T(I,2)='EO'                                                C  002:01C1:2
      T(I,3)='LOW '                                              C  002:01C4:1
      I=I+1                                                      C  002:01C7:1
      T(I,2)='EO'                                                C  002:01C8:3
      T(I,3)='HIGH'                                              C  002:01CB:1
      I=I+1                                                      C  002:01CE:1
      T(I,2)='FO'                                                C  002:01CF:3
      T(I,3)='LOW '                                              C  002:01D2:1
      I=I+1                                                      C  002:01D5:1
      T(I,2)='FO'                                                C  002:01D6:3
      T(I,3)='HIGH'                                              C  002:01D9:1
      I=I+1                                                      C  002:01DC:1
      T(I,2)='EX'                                                C  002:01DD:3
      T(I,3)='LOW '                                              C  002:01E0:1
      I=I+1                                                      C  002:01E3:1
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      T(I,2)='EX'                                                          C 002:01E4:13
      T(I,3)='HIGH'                                                        C 002:01E7:11
      I=I+1                                                                C 002:01EA:11
      T(I,2)='FX'                                                          C 002:01EB:13
      T(I,3)='LOW '                                                        C 002:01EE:11
      I=I+1                                                                C 002:01F1:11
      T(I,2)='FX'                                                          C 002:01F2:13
      T(I,3)='HIGH'                                                        C 002:01F5:11
C         END LABEL ASSIGNMENT                                             C 002:01F8:11
C         NMAG = 1 FOR ORDINARY WAVE; NMAG = 2 FOR EXTRAORDINARY WAVE      C 002:01F8:11
      DO50NMAG=1,2                                                         C 002:01F8:11
      IF(NMAG-1)36,36,37                                                   C 002:01F9:10
C         SUBROUTINE 'PATH' RETURNS ARRAY P(I,J) CONTAINING SIGNAL         C 002:01FA:14
C         PATH LENGTH, DELAY AND DELAY DERIVATIVES                         C 002:01FA:14
   36 CALL PATH(RH,FPEO,FPFO,FC,P)                                         C 002:01FA:14
      GOTO38                                                               C 002:01FE:10
   37 CALL PATH(RH,FPEX,FPFX,FC,P)                                         C 002:01FE:13
   38 DO40I=1,4                                                            C 002:0201:15
      II=I+4*(NMAG-1)*NNHOP                                                C 002:0203:10
      T(II,4)=P(I,1)                                                       C 002:0205:15
      T(II,5)=(180./PI)*ARCOS(P(I,3))*P(I,1)                               C 002:0206:10
      T(II,6)=XNHOP*P(I,4)                                                 C 002:020D:13
      T(II,7)=P(I,5)*XNHOP                                                 C 002:0210:14
      T(II,8)=FC*P(I,5)*XNHOP*(1.E+06)                                     C 002:0214:10
      INT=T(II,8)                                                          C 002:0219:12
      T(II,8)=T(II,8)-FLOAT(INT)                                           C 002:021A:10
      T(II,9)=(P(I,5)+FC*P(I,6))*XNHOP                                     C 002:021C:13
      T(II,10)=(P(I,6)+.5*FC*P(I,7))*XNHOP                                 C 002:0221:14
      T(II,11)=((.5*P(I,7))+((FC*P(I,8))/6.))*XNHOP                        C 002:0226:14
C         COMPUTE ATTENUATION                                              C 002:0230:12
      XIND=P(I,1)-(1.E-08)                                                 C 002:0230:12
      IF(XIND)21,21,20                                                     C 002:0233:14
   20 TH=T(II,5)                                                           C 002:0234:13
      DIST=T(II,6)                                                         C 002:0236:11
      XMAG=3.-2.*FLOAT(NMAG)                                               C 002:0237:15
      A(1)=ARSMLT                                                          C 002:0239:14
      A(2)=SPIMLT                                                          C 002:023A:15
      A(3)=SOLEXP                                                          C 002:023C:10
      CALLATTEN(XLAIM,TH,RAM,SPT,ZEN,TIME,DIST,NHOP,FC,XMAG,A,ATT)         C 002:023D:12
      T(II,12)=ATT                                                         C 002:0244:13
      GOTO22                                                               C 002:0246:13
   21 T(II,12)=0.                                                          C 002:0247:10
C         END COMPUTATION OF ATTENUATION                                   C 002:0248:15
C         COMPUTE DOPPLER SHIFTS AND SPREADS                               C 002:0248:15
   22 XIND=FLOAT(I)-2.5                                                    C 002:0248:15
      IF(XIND)23,23,24                                                     C 002:0248:14
   23 T(II,13)=XNHOP*SHIFTE*((FC/RFREQE)**FXSHTE)*P(I,1)                   C 002:024C:13
      T(II,14)=SQRT(XNHOP)*SPRDE*((FC/RFREQE)**EXSPRE)*P(I,1)              C 002:0252:13
      GOTO25                                                               C 002:0259:11
   24 T(II,13)=XNHOP*SHIFTF*((FC/RFREQF)**FXSHTF)*P(I,1)                   C 002:0259:14
      T(II,14)=SQRT(XNHOP)*SPRDF*((FC/RFREQF)**EXSPRF)*P(I,1)              C 002:025F:14
   25 CONTINUE                                                             C 002:0260:12
C         END COMPUTATION OF SHIFTS AND SPREADS                            C 002:0260:12
   40 CONTINUE                                                             C 002:0260:12
   50 CONTINUE                                                             C 002:0260:13
   60 CONTINUE                                                             C 002:026A:14
C         DO 70 DETERMINES MINIMUM (NONZERO) ATTENUATION                   C 002:026C:15
      ATTMIN=1.E+05                                                        C 002:026C:15
      ATTMAX=0.                                                            C 002:026E:13
      DO70I=1,48                                                           C 002:026F:11
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      IF(T(I,12))70,70,61                                              C 002027010
 61   ATTMIN=AMIN1(T(I,12),ATTMIN)                                     C 002027211
      ATTMAX=AMAX1(T(I,12),ATTMAX)                                     C 002027010
 70   CONTINUE                                                         C 002027015
      IF(ATTMAX)71,71,72                                               C 002027010
 71   WRITE(6,950)                                                     C 002027011
      NPATH=0                                                          C 002028012
      GOTO120                                                          C 002028010
 72   WRITE(6,938)                                                     C 002028213
      WRITE(6,939)                                                     C 002028012
      WRITE(6,940)                                                     C 002028A12
      IF(IPRINT.IS.PRET)GOTO101                                        C 002028012
C          DO 100 PRINTS THE COMPLETE ARRAY T(I,J)                     C 002028014
      DO100I=1,48                                                      C 002028014
      IN=1(T(I,1)+1.E-05                                               C 002029010
      WRITE(6,941)IN,T(I,2),T(I,3),T(I,4),T(I,5),T(I,6),T(I,7),T(I,8),T(  C 002029015
     1I,9),T(I,10),T(I,11),T(I,12),T(I,13),T(I,14)                    C 00202AA11
100   CONTINUE                                                         C 00202BB12
C          DO 110 DETERMINES NPATH AND MODIFIES ARRAY T(I,J) SO THAT  C 00202BB13
C          IT CONTAINS ONLY THE DESIRED RETURNS                       C 00202BB13
101   ATTLIM=ATTMIN+ATTDIF                                             C 00202BB13
      NPATH=0                                                          C 00202BE15
      DO110I=1,48                                                      C 00202BF13
      IF(T(I,12))110,110,102                                           C 00202C110
102   IF(T(I,12)-ATTLIM)103,103,110                                    C 00202C311
103   NPATH=NPATH+1                                                    C 00202C513
      DO105J=1,14                                                      C 00202C615
      T(NPATH,J)=T(I,J)                                                C 00202C810
105   CONTINUE                                                         C 00202CC14
110   CONTINUE                                                         C 00202CE15
      IF(IPRINT.IS.PRET)GOTO114                                        C 00202D110
      GOTO115                                                          C 00202D212
C          DO 115 PRINTS THE MODIFIED ARRAY T(I,J)                     C 00202D215
114   DO115I=1,NPATH                                                   C 00202D215
      IN=1(T(I,1)+1.E-05                                               C 00202D410
      WRITE(6,951)IN,T(I,2),T(I,3),T(I,5),T(I,6),T(I,7),T(I,8),T(I,9),T(  C 00202D715
     1I,10),T(I,11),T(I,12),T(I,13),T(I,14)                           C 00202E012
115   CONTINUE                                                         C 00202F012
      WRITE(6,900)                                                     C 00202F113
      WRITE(6,952)ATTDIF                                               C 002030212
120   CONTINUE                                                         C 002030012
C          PUNCH DATA CARDS FOR USE IN SIGNAL ANALYSIS PART OF SIMULATOR  C 002030012
C          IF NPUNCH=1 (PUNCHES MODIFIED ARRAY T(I,J))                 C 002030012
      IF(NPUNCH-1)140,121,140                                          C 002030012
121   PUNCH/,,CARN=,NPATH                                              C 002030A13
                                                                          FIB IS 0006 LONG
      PUNCH 909,TIME                                                   C 002031012
      PUNCH/,,DELAY,GATTEN                                             C 002031012
      DO130I=1,NPATH                                                   C 002032012
      PUNCH 935,(T(I,J),J=1,7)                                         C 002032010
      PUNCH 936,(T(I,J),J=8,11)                                        C 002033012
      PUNCH 937,(T(I,J),J=12,14)                                       C 002034012
130   CONTINUE                                                         C 002034012
140   CONTINUE                                                         C 002034013
C          END PUNCH DATA CARDS                                        C 002034013
900   FORMAT(1X)                                                       C 002034013
901   FORMAT(1X,/)                                                     C 002034013
902   FORMAT(1X,//)                                                    C 002034013
903   FORMAT(1X,///)                                                   C 002034013
904   FORMAT(1X,////)                                                  C 002034013
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909  FORMAT(A6)                                                          C  0021034F13
910  FORMAT(46X,'SUMMARY OF TRANSMISSION PARAMETERS'///)                 C  0021034F13
911  FORMAT(1X,'SIGNAL PARAMETERS'/)                                     C  0021034F13
912  FORMAT(9X,'CARRIER FREQ. =',F7.3,' MHZ,     BANDWIDTH =',F7.2,' KHZ C  0021034F13
    1.'/)                                                                C  0021034F13
913  FORMAT(1X,'TRANSMITTER/RECEIVER PARAMETERS'/)                       C  0021034F13
915  FORMAT(9X,'LOCATION OF TRANSMITTER: LONGITUDE = ',F7.2,' DEGREES (  C  0021034F13
    1+ FOR WEST, - FOR EAST)')                                           C  0021034F13
916  FORMAT(34X,'LATITUDE  = ',F7.2,' DEGREES (+ FOR NORTH, - FOR SOUTH  C  0021034F13
    1)')                                                                 C  0021034F13
917  FORMAT(9X,'DISTANCE BETWEEN TRANSMITTER AND RECEIVER =',F8.2,' KM.  C  0021034F13
    1')                                                                  C  0021034F13
918  FORMAT(9X,'DIRECTION FROM TRANSMITTER TO RECEIVER =',F6.2,' DEGREE  C  0021034F13
    1S (MEASURED FROM TRUE NORTH)')                                      C  0021034F13
919  FORMAT(9X,'TRANSMITTER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)')    C  0021034F13
920  FORMAT(9X,'RECEIVER ANTENNA PATTERN = (3/2)*(SIN(THETA)**2)')       C  0021034F13
921  FORMAT(9X,'SEA STATE =',I2/)                                       C  0021034F13
922  FORMAT(1X,'IONOSPHERE PARAMETERS'/)                                 C  0021034F13
923  FORMAT(9X,'SUNSPOT NUMBER = ',F7.2)                                 C  0021034F13
924  FORMAT(9X,'DAYTIME ELECTRON DENSITY PROFILE')                       C  0021034F13
925  FORMAT(9X,'NIGHTTIME ELECTRON DENSITY PROFILE')                     C  0021034F13
926  FORMAT(15X,'E REGION: HEIGHT OF MAX. ELECTRON DENSITY =',F7.2,' KM  C  0021034F13
    1.')                                                                 C  0021034F13
927  FORMAT(25X,'SEMITHICKNESS OF PARABOLA =',F7.2,' KM.')               C  0021034F13
928  FORMAT(25X,'PENETRATION FREQUENCY, ORDINARY WAVE =',F7.3,' MHZ.')   C  0021034F13
929  FORMAT(15X,'F REGION: HEIGHT OF MAX. ELECTRON DENSITY =',F7.2,' KM  C  0021034F13
    1.')                                                                 C  0021034F13
930  FORMAT(25X,'PENETRATION FREQUENCY, EXTRAORDINARY WAVE =',F7.3,' MH  C  0021034F13
    1Z.')                                                                C  0021034F13
932  FORMAT(9X,'SOLAR ZENITH ANGLE =',F7.2,' DEGREES')                   C  0021034F13
933  FORMAT(9X,'SOLAR ZENITH ANGLE = NOT APPLICABLE TO NIGHTTIME TRANSM  C  0021034F13
    1ISSION')                                                            C  0021034F13
934  FORMAT(1X,'COMPUTED CHANNEL PARAMETERS'/)                           C  0021034F13
935  FORMAT(1X,'GROUNDWAVE: DELAY = ',1PE10.3,' SEC.     ATTENUATION =   C  0021034F13
    1',E10.3,' DB.')                                                     C  0021034F13
936  FORMAT(1X,'IONOSPHERE RETURNS'/)                                    C  0021034F13
938  FORMAT( 3X,'MODE',3X,'SOLUTION',3X,'RAY ANGLE',3X,'PATH',4X,'CARRI  C  0021034F13
    1ER',4X,'CARRIER',3X,'  SIGNAL  ',1X,'AMPLITUDE',4X,'PHASE',5X,'ATT  C  0021034F13
    2ENUATION',2X,'DOPPLER',4X,'DOPPLER')                                C  0021034F13
939  FORMAT(10X,'INDICATOR',2X,'(DEGREES)',2X,'LENGTH',4X,'DELAY',6X,'P  C  0021034F13
    1HASE',6X,'DELAY',3X,'DISTORTION',1X,' DISTORTION',5X,'(DB)', 7X,'S  C  0021034F13
    2HIFT',5X,'SPREAD')                                                  C  0021034F13
940  FORMAT( 9X,'(0=NO SOLN)',13X,'(KM)',5X,'(SEC)',4X,'(CYCLES)',5X,'(  C  0021034F13
    1SEC)',4X,'(SEC/MHZ)',1X,'(SEC/MHZ**2)',15X,'(HZ)', 7X,'(HZ)'/)      C  0021034F13
941  FORMAT(1X,I1,A2,1X,A4,4X,F3.1,7X,F5.2,1X,F9.2,1X,1PE10.3,1X,E10.3,  C  0021034F13
    11X,E10.3,1X,F10.3,2X,E10.3,2X,E10.3,1X,E10.3,1X,E10.3)              C  0021034F13
945  FORMAT(9X,'DOPPLER REFERENCE VALUES')                               C  0021034F13
946  FORMAT(15X,'E REGION: SHIFT =',F8.4,' HZ,',1X,'SHIFT EXP. =',F6.3,  C  0021034F13
    1'; SPREAD =',F8.4,' HZ,',1X,'SPREAD EXP. =',F6.3,'; REF. FREQ. =',  C  0021034F13
    2F7.3,' MHZ.')                                                       C  0021034F13
947  FORMAT(15X,'F REGION: SHIFT =',F8.4,' HZ,',1X,'SHIFT EXP. =',F6.3,  C  0021034F13
    1'; SPREAD =',F8.4,' HZ,',1X,'SPREAD EXP. =',F6.3,'; REF. FREQ. =',  C  0021034F13
    2F7.3,' MHZ.')                                                       C  0021034F13
950  FORMAT(22X,'***** THERE ARE NO RETURNS FROM THE IONOSPHERE FOR THI  C  0021034F13
    1S FREQUENCY AND RANGE *****')                                       C  0021034F13
951  FORMAT(1X,I1,A2,1X,A4,4X,'NA',8X,F5.2,1X,F9.2,1X,1PE10.3,1X,E10.3,  C  0021034F13
    11X,E10.3,1X,F10.3,2X,E10.3,2X,E10.3,1X,E10.3,1X,E10.3)              C  0021034F13
952  FORMAT(9X,'ALL IONOSPHERE RETURNS WHOSE ATTENUATION IS GREATER THA  C  0021034F13
    1N ',F7.2,' DB. ABOVE THE MINIMUM ATTENUATION ARE IGNORED')          C  0021034F13
953  FORMAT(9X,'NONDEVIATIVE ABSORPTION CONSTANTS')                      C  0021034F13
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954  FORMAT(15X,'MULTIPLICATIVE CONSTANT =',F8.2,', SUNSPOT NUMBER MULT          C  002:034F:3
    1IPLIER =',F9.6,', SOLAR ZENITH ANGLE EXPONENT =',F6.3)                      C  002:034F:3
955  FORMAT(I1,A2,A4,4E16.9)                                                     C  002:034F:3
956  FORMAT(4E16.9)                                                              C  002:034F:3
957  FORMAT(3E16.9)                                                              C  002:034F:3
     STOP                                                                        C  002:034F:3
     END                                                                         C  002:0350:2
     002:0352:3   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:00AD
     002:0353:5   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:009C
     002:0355:1   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0094
     002:0356:3   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0084
     002:0357:5   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0074
     002:0359:1   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0068
     002:035A:3   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:005A
     002:035B:5   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:004D
     002:035D:1   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:003D
     002:035E:3   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0035
     002:035F:5   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0020
     002:0361:1   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0025
     002:0362:3   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0010
     002:0363:5   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0013
     002:0365:1   IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0009
                                                                   SEGMENT 002 IS 03CF LONG
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                                                                 START OF SEGMENT 00B
      SUBROUTINE BNGMAG(PHIT,XLAMT,R,ANGLEG,ANGLEM)              C 00B:000010
                                                                 C 00B:000010
C          SUBROUTINE COMPUTES BEARING OF THE RECEIVER FROM THE  C 00B:000010
C          TRANSMITTER IN MAGNETIC COORDINATES                   C 00B:000010
C             PHIT  = GEOGRAPHIC LATITUDE OF TRANSMITTER (DEGREES)   C 00B:000010
C             XLAMT = GEOGRAPHIC LONGITUDE OF TRANSMITTER (DEGREES)  C 00B:000010
C             R     = DISTANCE BETWEEN TRANSMITTER AND RECEIVER (KM.)  C 00B:000010
C             ANGLEG= BEARING OF RECEIVER IN GEOGRAPHIC COORD. (DEGREES)  C 00B:000010
C             ANGLEM= BEARING OF RECEIVER IN MAGNETIC COORD. (DEGREES)    C 00B:000010
                                                                 C 00B:000010
      PI=3.14159265359                                           C 00B:000010
      CC=PI/180.                                                 C 00B:000510
      GG=ANGLEG/180.                                             C 00B:000610
      IF(GG)6,5,6                                                C 00B:000712
5     PHIR=PHIT+(R/(CC*6370.))                                   C 00B:000811
      GOTO8                                                      C 00B:000A14
6     IF(GG-1.)10,7,10                                           C 00B:000B11
7     PHIR=PHIT-(R/(CC*6370.))                                   C 00B:000C12
8     XLAMR=XLAMT                                                C 00B:000E15
      GOTO20                                                     C 00B:000F14
10    A=CC*(90.-PHIT)                                            C 00B:001011
      B=R/6370.                                                  C 00B:001210
      G=(CC*ANGLEG)/2.                                           C 00B:001313
      D1=(A-B)/2.                                                C 00B:001512
      S1=(A+B)/2.                                                C 00B:001711
      W1=SIN(D1)/(TAN(G)*SIN(S1))                                C 00B:001910
      W2=COS(D1)/(TAN(G)*COS(S1))                                C 00B:001C15
      Z1=ATAN(W1)                                                C 00B:002014
      Z2=ATAN(W2)                                                C 00B:002211
      W3=(TAN(S1)*COS(Z2))/COS(Z1)                               C 00B:002314
      C=2.*ATAN(W3)                                              C 00B:002713
      PHIR=90.-(C/CC)                                            C 00B:002913
      XLAMR=XLAMT+((Z2-Z1)/CC)                                   C 00B:002B12
20    CALL GEIOMA(PHIT,XLAMT,PHITM,XLAMTM)                       C 00B:002D14
      CALL GEIOMA(PHIR,XLAMR,PHIRM,XLAMRM)                       C 00B:003012
      D2=(CC/2.)*(PHIRM-PHITM)                                   C 00B:003310
      S2=(CC/2.)*(PHIRM+PHITM)                                   C 00B:003512
      D=(CC/2.)*(XLAMTM-XLAMRM)                                  C 00B:003714
      IF(D)55,50,55                                              C 00B:003A10
50    IF(PHIRM-PHITM)51,52,52                                    C 00B:003A15
51    ANGLEM=180.                                                C 00B:003C11
      GOTO100                                                    C 00B:003D10
52    ANGLEM=0.                                                  C 00B:003D13
      GOTO100                                                    C 00B:003E11
55    W4=SIN(D2)/(TAN(D)*COS(S2))                                C 00B:003E14
      W5=COS(D2)/(TAN(D)*SIN(S2))                                C 00B:004213
      Z4=ATAN(W4)                                                C 00B:004612
      Z5=ATAN(W5)                                                C 00B:004715
      ANGLEM=((Z5-Z4)/CC)                                        C 00B:004912
100   RETURN                                                     C 00B:004A15
      END                                                        C 00B:004B12
                                                      SEGMENT 00B IS 005C LONG
```

```
                                                                           START OF SEGMENT OOF
      SUBROUTINE GETOMA(PHIG,XLAMG,PHIM,XLAMM)                         C  OOF:000010
                                                                       C  OOF:000010
C           SUBROUTINE CONVERTS GEOGRAPHIC COORD. TO MAGNETIC COORD.   C  OOF:000010
C              PHIG  = GEOGRAPHIC LATITUDE (DEGREES)                   C  OOF:000010
C              XLAMG = GEOGRAPHIC LONGITUDE (DEGREES)                  C  OOF:000010
C              PHIM  = MAGNETIC LATITUDE (DEGREES)                     C  OOF:000010
C              XLAMM = MAGNETIC LONGITUDE (DEGREES)                    C  OOF:000010
                                                                       C  OOF:000010
      PI=3.14159265359                                                 C  OOF:000010
      C=PI/180.                                                        C  OOF:000510
      A1=C*PHIG                                                        C  OOF:000610
      A2=C*XLAMG                                                       C  OOF:000710
      A3=C*78.3                                                        C  OOF:000812
      A4=C*69.                                                         C  OOF:000A14
      B1=SIN(A1)*SIN(A3)+COS(A1)*COS(A3)*COS(A2-A4)                    C  OOF:000C10
      PHIM=ARSIN(B1)                                                   C  OOF:001214
      B2=(COS(A1)*SIN(A2-A4))/COS(PHIM)                                C  OOF:001411
      PHIM=PHIM/C                                                      C  OOF:001813
      XLAMM=ARSIN(B2)/C                                                C  OOF:001915
      RETURN                                                           C  OOF:001B15
      END                                                              C  OOF:001C12
                                                                   SEGMENT OOF IS 0028 LONG
```

```
                                                                    START OF SEGMENT 011
      SUBROUTINEATTEN(XLATH,TH,RAM,SPT,ZEN,TIME,DIST,NHOP,FC,XMAG,A,ATT)  C  011:000010
      DIMENSION A(3)                                                       C  011:000010
C                                                                          C  011:000010
C          SUBROUTINE COMPUTES TOTAL ATTENUATION OF IONOSPHERE RETURNS    C  011:000010
C              XLATH  = MAGNETIC LATITUDE (DEGREES)                        C  011:000010
C              TH     = RAY ANGLE MEASURED FROM VERTICAL (DEGREES)         C  011:000010
C              RAM    = BEARING OF RECEIVER IN MAGNETIC COORD. (DEGREES)   C  011:000010
C              SPT    = SUNSPOT NUMBER                                     C  011:000010
C              ZEN    = SOLAR ZENITH ANGLE (DEGREES)                       C  011:000010
C              TIME   = TIME OF DAY ('DAY' OR 'NITE')                      C  011:000010
C              NHOP   = NUMBER OF HOPS                                     C  011:000010
C              DIST   = TOTAL SIGNAL PATH LENGTH (KM.)                     C  011:000010
C              FC     = CARRIER FREQUENCY (MHZ.)                           C  011:000010
C              XMAG   = INDICATOR (+1 FOR O-WAVE, -1 FOR X-WAVE)           C  011:000010
C              A(1)   = MULTIPLICATIVE CONST. FOR NONDEVIATIVE ABSORPTION  C  011:000010
C              A(2)   = SUNSPOT NO. MULTIPLIER FOR NONDEVIATVE ABSORPTION  C  011:000010
C              A(3)   = SOLAR ZEN. ANG. EXPNT. FOR NONDEVIATVE ABSORPTION  C  011:000010
C              ATT    = TOTAL SIGNAL ATTENUATION                           C  011:000010
C                                                                          C  011:000010
      PI=3.14159265359                                                     C  011:000010
      CC=PI/180.                                                           C  011:000510
      TN='NITE'                                                            C  011:000610
      XTH=CC*TH                                                            C  011:000813
C          COMPUTE ABSORPTION IN D REGION                                  C  011:000915
      IF(TIME.IS.TN)GOTO9                                                  C  011:000915
      ANGIND=ATAN(-2.*TAN(CC*XLATH))                                       C  011:000B11
      XRAM=CC*RAM                                                          C  011:000E13
      XA=SIN(XTH)*COS(XRAM)*COS(ANGIND)                                    C  011:000F15
      XB=COS(XTH)*SIN(ANGIND)                                              C  011:001314
      XI=ABS(XA+XB)                                                        C  011:001612
      XR=ABS(XA-XB)                                                        C  011:001810
      FH=.87*SQRT(1.+3.*((SIN(CC*XLATH))**2))*((6370./6440.)**3)           C  011:001914
      B=(A(1)*(1.+A(2)*SPT)*((COS(CC*ZEN))**A(3)))/COS(XTH)                C  011:002110
      DBI=B/((FC+XMAG*FH*XI)**2)                                           C  011:002911
      DBR=B/((FC+XMAG*FH*XR)**2)                                           C  011:002C10
      DBAB=FLOAT(NHOP)*(DBI+DBR)                                           C  011:002F11
      GOTO10                                                               C  011:003110
    9 DBAB=0.                                                              C  011:003113
   10 CONTINUE                                                             C  011:003211
C          END COMPUTATION OF ABSORPTION                                   C  011:003211
C          COMPUTE 1/R**2 LOSS                                             C  011:003211
      DBR2=20.*ALOG10((4.*PI*DIST*FC)/.3)                                  C  011:003211
C          END COMPUTATION OF 1/R**2 LOSS                                  C  011:003710
C          COMPUTE ANTENNA PATTERN LOSS                                    C  011:003710
      RTH=SQRT(3.)/2.                                                      C  011:003710
      DBANT=-40.*ALOG10(RTH*SIN(XTH))                                      C  011:003910
C          END COMPUTATION OF ANTENNA LOSS                                 C  011:003C12
      ATT=DBAB+DBR2+DBANT                                                  C  011:003C12
      RETURN                                                               C  011:003E11
      END                                                                  C  011:003E14
                                                                    SEGMENT 011 IS 0052 LONG
```

```
                                                                        START OF SEGMENT 013
      SUBROUTINE PATH(R,FPE,FPF,F,P)                                    C 013:000010
      COMMON HE,HEM,HF,HFM,TIME                                         C 013:000010
      DIMENSION P(4,8)                                                  C 013:000010
                                                                        C 013:000010
C          EVALUATES ANGLE, PATH DISTANCE,DELAY AND DELAY DERIVATIVES   C 013:000010
C          FOR E REGION AND F REGION                                    C 013:000010
C              4HE = THICKNESS OF E REGION (KM.)                        C 013:000010
C              HEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)  C 013:000010
C              FPE = PENETRATION FREQUENCY OF E REGION (MHZ.)           C 013:000010
C              4HF = THICKNESS OF F REGION (KM.)                        C 013:000010
C              HFM = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.)  C 013:000010
C              FPF = PENETRATION FREQUENCY OF F REGION (MHZ.)           C 013:000010
C              TIME= TIME OF DAY ('DAY' OR 'NITE')                      C 013:000010
C              R   = LENGTH OF GROUND PATH (KM.)                        C 013:000010
C              F   = FREQUENCY (MHZ.)                                   C 013:000010
C              X   = COS(THETA); THETA=RAY ANGLE MEASURED FROM VERTICAL C 013:000010
C          SUBROUTINE RETURNS P(I,J)                                    C 013:000010
C              I = 1: LOW  RAY. F REGION                                C 013:000010
C                = 2: HIGH RAY. F REGION                                C 013:000010
C                = 3: LOW  RAY. F REGION                                C 013:000010
C                = 4: HIGH RAY. F REGION                                C 013:000010
C              J = 1: SOLUTION INDICATOR (0=NO SOLUTION, 1=SOLUTION)    C 013:000010
C                = 2: R=LENGTH OF GROUND PATH (KM.)                     C 013:000010
C                = 3: X=COS(THETA)                                      C 013:000010
C                = 4: D=LENGTH OF SIGNAL PATH (KM.)                     C 013:000010
C                = 5: T=PATH DELAY (SEC.)                               C 013:000010
C                = 6: D1T=DERIVATIVE OF DELAY (SEC./MHZ.)               C 013:000010
C                = 7: D2T=SECOND DERIVATIVE OF DELAY (SEC./MHZ.**2)     C 013:000010
C                = 8: D3T=THIRD DERIVATIVE OF DELAY (SEC./MHZ.**3)      C 013:000010
                                                                        C 013:000010
      FNE=F/FPE                                                         C 013:000010
      FNF=F/FPF                                                         C 013:000112
      IF(FNE-1.)10,10,20                                                C 013:000214
   10 XMIN=1.                                                           C 013:000315
      DO15J=1,8                                                         C 013:000413
      P(2,J)=0.                                                         C 013:000610
      P(3,J)=0.                                                         C 013:000614
      P(4,J)=0.                                                         C 013:000612
   15 CONTINUE                                                          C 013:000615
      GOTO40                                                            C 013:001010
                                                                        C 013:001013
C          DO 30 DETERMINES MINIMUM VALUE OF AE(X)                      C 013:001013
                                                                        C 013:001013
   20 XLAST=1.E-10                                                      C 013:001013
      CALL AE(FPE,F,XLAST,ALAST)                                        C 013:001213
      DO30L=1,9                                                         C 013:001511
      DIFF=1./(10.**L)                                                  C 013:001610
   26 XMIN=XLAST+DIFF                                                   C 013:001613
      IF(FNE*XMIN-1.)27,29,29                                           C 013:001915
   27 CALL AE(FPE,F,XMIN,AMIN)                                          C 013:001613
      IF(AMIN-ALAST)28,29,29                                            C 013:001E11
   28 XLAST=XMIN                                                        C 013:001F13
      ALAST=AMIN                                                        C 013:002012
      GOTO26                                                            C 013:002111
   29 IF(L-1)36,36,37                                                   C 013:002114
   36 XLAST=XMIN-DIFF                                                   C 013:002215
      GOTO38                                                            C 013:002411
   37 XLAST=XMIN-2.*DIFF                                                C 013:002414
   38 CALL AE(FPE,F,XLAST,ALAST)                                        C 013:002613
```

```
30    CONTINUE                                                     C 013:002B11
C                                                                  C 013:002B12
C          IF MIN. AE(X) .GT. P, THERE ARE NO RETURNS FROM E REGION C 013:002B12
C                                                                  C 013:002B12
      IF(AMIN-R)40,40,31                                           C 013:002B12
31    DO35J=1,8                                                    C 013:002C14
      P(1,J)=0.                                                    C 013:002E10
      P(2,J)=0.                                                    C 013:003014
35    CONTINUE                                                     C 013:003312
      GOTO80                                                       C 013:003513
C                                                                  C 013:003610
C          DO 50 DETERMINES X = COS(THETA) FOR E REGION LOW RAY    C 013:003610
C                                                                  C 013:003610
40    XLAST=1.E-10                                                 C 013:003610
      DO50L=1,8                                                    C 013:003613
      DIFF=1./(10.**L)                                             C 013:003A10
46    X=XLAST+DIFF                                                 C 013:003C13
      IF(X-XMIN)47,49,49                                           C 013:003D15
47    CALL AE(FPE,F,X,A)                                           C 013:003E15
      IF(A-R)49,49,48                                              C 013:004113
48    XLAST=X                                                      C 013:004215
      GOTO46                                                       C 013:004314
49    XLAST=X-DIFF                                                 C 013:004411
50    CONTINUE                                                     C 013:004513
      P(1,1)=1.                                                    C 013:004714
      P(1,2)=R                                                     C 013:004815
      X=AMIN1(X,1.)                                                C 013:004A14
      P(1,3)=X                                                     C 013:004D12
      P(1,4)=H/SQRT(1.-X**2)                                       C 013:004F11
      CALL DELAYE(R,FPE,F,X,T,D1T,D2T,D3T)                         C 013:005215
      P(1,5)=T                                                     C 013:005713
      P(1,6)=D1T                                                   C 013:005912
      P(1,7)=D2T                                                   C 013:005B11
      P(1,8)=D3T                                                   C 013:005D10
      IF(FNE-1.)200,200,60                                         C 013:005E15
C                                                                  C 013:006013
C          DO 70 DETERMINES X = COS(THETA) FOR E REGION HIGH RAY   C 013:006013
C                                                                  C 013:006013
60    XLAST=(1./FNE)-1.E-10                                        C 013:006013
      DO70L=1,9                                                    C 013:006314
      DIFF=1./(10.**L)                                             C 013:006510
66    X=XLAST-DIFF                                                 C 013:006713
      IF(X-XMIN)69,69,67                                           C 013:006815
67    CALL AE(FPE,F,X,A)                                           C 013:006915
      IF(A-R)69,69,68                                              C 013:006C13
68    XLAST=X                                                      C 013:006D15
      GOTO66                                                       C 013:006E14
69    XLAST=X+DIFF                                                 C 013:006F11
70    CONTINUE                                                     C 013:007013
      P(2,1)=1.                                                    C 013:007214
      P(2,2)=H                                                     C 013:007411
      X=AMIN1(X,1.)                                                C 013:007610
      P(2,3)=X                                                     C 013:007814
      P(2,4)=H/SQRT(1.-X**2)                                       C 013:007A13
      CALL DELAYE(H,FPE,F,X,T,D1T,D2T,D3T)                         C 013:007E11
      P(2,5)=T                                                     C 013:008215
      P(2,6)=D1T                                                   C 013:008414
      P(2,7)=D2T                                                   C 013:008613
      P(2,8)=D3T                                                   C 013:008812
80    IF(FNF-1.)90,90,100                                          C 013:008A11
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   90   XMIN=1.                                                          C  013100B612
        DO95J=1,8                                                        C  013100BC10
        P(4,J)=0.                                                        C  013100BD10
   95   CONTINUE                                                         C  013100BF13
        GOTO120                                                          C  0131009114
                                                                         C  0131009211
C            DO 110 DETERMINES MINIMUM VALUE OF AF(X)                    C  0131009211
                                                                         C  0131009211
  100   XLAST=(1./FNF)+1.E-11                                            C  0131009211
        CALL AF(FPE,FPF,F,XLAST,ALAST)                                   C  0131009414
        DO110L=1,10                                                      C  0131009715
        DIFF=1./(10.**L)                                                 C  0131009910
  106   XMIN=XLAST+DIFF                                                  C  0131009B13
        IF(FNF*XMIN-1.)107,109,109                                       C  0131009C15
  107   CALL AF(FPE,FPF,F,XMIN,AMIN)                                     C  0131009E13
        IF(AMIN-ALAST)108,109,109                                        C  013100A114
  108   XLAST=XMIN                                                       C  013100A310
        ALAST=AMIN                                                       C  013100A315
        GOTO106                                                          C  013100A414
  109   IF(L-1)116,116,117                                               C  013100A511
  116   XLAST=XMIN-DIFF                                                  C  013100A612
        GOTO118                                                          C  013100A714
  117   XLAST=XMIN-2.*DIFF                                               C  013100A811
  118   CALL AF(FPE,FPF,F,XLAST,ALAST)                                   C  013100AA10
  110   CONTINUE                                                         C  013100AD11
                                                                         C  013100AF12
C            IF MIN. AF(X) .GT. R, THERE ARE NO RETURNS FROM F REGION    C  013100AF12
                                                                         C  013100AF12
        IF(AMIN-R)120,120,111                                            C  013100AF12
  111   DO115J=1,8                                                       C  013100B014
        P(3,J)=0.                                                        C  013100B210
        P(4,J)=0.                                                        C  013100B414
  115   CONTINUE                                                         C  013100B711
        GOTO200                                                          C  013100B912
                                                                         C  013100B915
C            DO 130 DETERMINES X = COS(THETA) FOR F REGION LOW RAY       C  013100B915
                                                                         C  013100B915
  120   XLAST=(1./FNF)+1.E-11                                            C  013100B915
        DO130L=1,9                                                       C  013100BC14
        DIFF=1./(10.**L)                                                 C  013100BE10
  126   X=XLAST+DIFF                                                     C  013100C013
        IF(X-XMIN)127,129,129                                            C  013100C115
  127   CALL AF(FPE,FPF,F,X,A)                                           C  013100C215
        IF(A-R)129,129,128                                               C  013100C610
  128   XLAST=X                                                          C  013100C712
        GOTO126                                                          C  013100C811
  129   XLAST=X-DIFF                                                     C  013100C814
  130   CONTINUE                                                         C  013100CA10
        P(3,1)=1.                                                        C  013100CC11
        P(3,2)=R                                                         C  013100CD15
        X=AMIN1(X,1.)                                                    C  013100CF14
        P(3,3)=X                                                         C  013100D212
        P(3,4)=R/SQRT(1.-X**2)                                           C  013100D411
        CALL DELAYF(R,FPE,FPF,F,X,T,D1T,D2T,D3T)                         C  013100D715
        P(3,5)=T                                                         C  013100DD10
        P(3,6)=D1T                                                       C  013100DE15
        P(3,7)=D2T                                                       C  013100E014
        P(3,8)=D3T                                                       C  013100E213
        IF(FNF-1.)200,200,140                                            C  013100E412
                                                                         C  013100E610
```

```
C          DO 150 DETERMINES X = COS(THETA) FOR F REGION HIGH RAY          L  013:00E610
                                                                           L  013:00E610
  140   XLAST=(1./FNF)-1.E-11                                              L  013:00E610
        DO150L=1,10                                                        L  013:00E614
        DIFF=1./(10.**L)                                                   L  013:00EA10
  146   X=XLAST-DIFF                                                       L  013:00EC13
        IF(X-XMIN)149,149,147                                              L  013:00ED15
  147   CALL AF(FPE,FPF,F,X,A)                                             L  013:00EE15
        IF(A-R)149,149,148                                                 L  013:00F210
  148   XLAST=X                                                            L  013:00F312
        GOTO146                                                            L  013:00F411
  149   XLAST=X+DIFF                                                       L  013:00F414
  150   CONTINUE                                                           L  013:00F610
        P(4,1)=1.                                                          L  013:00F811
        P(4,2)=H                                                           L  013:00F915
        X=AMIN1(X,1.)                                                      L  013:00FB14
        P(4,3)=X                                                           L  013:00FE12
        P(4,4)=H/SQRT(1.-X**2)                                             L  013:010011
        CALL DELAYF(R,FPE,FPF,F,Y,T,D1T,D2T,D3T)                           L  013:010315
        P(4,5)=T                                                           L  013:010910
        P(4,6)=D1T                                                         L  013:010A15
        P(4,7)=D2T                                                         L  013:010C14
        P(4,8)=D3T                                                         L  013:010E13
  900   RETURN                                                             L  013:011012
        END                                                                L  013:011015
                                                              SEGMENT 013 IS 011F LONG
```

```
                                                                   START OF SEGMENT 014
      SUBROUTINE AF(FPE,F,X,A)                                    C  0141000010
      COMMON HE,HEM,HF,HFM,TIME                                   C  0141000010
                                                                  C  0141000010
C         EVALUATES A(X)=A FOR THE E REGION                       C  0141000010
C              DHE  = THICKNESS OF E REGION (KM.)                 C  0141000010
C              HEM  = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)   C  0141000010
C              FPE  = PENETRATION FREQUENCY OF E REGION (MHZ.)    C  0141000010
C              X    = COS(THETA); THETA=RAY ANGLE MEASURED FROM VERTICAL  C  0141000010
C         WARNING: F*X MUST BE LESS THAN FPE                      C  0141000010
                                                                  C  0141000010
      FFE=F*X/FPE                                                 C  0141000010
      A=2.*SQRT((1./X**2)-1.)*(HEM-2.*HE+HF*FFE*ALOG((1.+FFE)/(1.-FFE)))  C  0141000115
      RETURN                                                      C  0141000A11
      END                                                         C  0141000A14
                                                              SEGMENT 014 IS 0014 LONG
```

```
                                                                        START OF SEGMENT 016
      SUBROUTINE AF(FPE,FPF,F,X,A)                                      C  016:000010
      COMMONHE,HEM,HF,HFM,TIME                                          C  016:000010
                                                                        C  016:000010
C        EVALUATES A(X)=A FOR THE F REGION                              C  016:000010
C           4HE  = THICKNESS OF E REGION (KM.)                          C  016:000010
C           HEM  = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)    C  016:000010
C           FPE  = PENETRATION FREQUENCY OF E REGION (MHZ.)             C  016:000010
C           4HF  = THICKNESS OF F REGION (KM.)                          C  016:000010
C           HFM  = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.)    C  016:000010
C           FPF  = PENETRATION FREQUENCY OF F REGION (MHZ.)             C  016:000010
C           TIME = TIME OF DAY ('DAY' OR 'NITE')                        C  016:000010
C           F    = FREQUENCY (MHZ.)                                     C  016:000010
C           X    = COS(THETA); THETA=RAY ANGLE MEASURED FROM VERTICAL   C  016:000010
C        WARNING: F*X MUST BE GREATER THAN FPE AND LESS THAN FPF        C  016:000010
                                                                        C  016:000010
      TN='NITE'                                                         C  016:000010
      FFE=F*X/FPE                                                       C  016:000213
      FFF=F*X/FPF                                                       C  016:000412
      FFF2=FFF**2                                                       C  016:000611
      FPR=(FPE**2)/(FPF**2)                                             C  016:000710
      IF(TIME.IS.TN)GOTO10                                              C  016:000910
      A=HEM-2.*HE+HE*FFE*ALOG((FFE+1.)/(FFE-1.))                        C  016:000A14
      A=A+(HFM-HEM-2.*HF*SQRT(1.-FPR))/SQRT(1.-(1./FFE**2))             C  016:001012
      A=A+2.*HF*FFF*ALOG((SQRT(1.-FPR)+SQRT(FFF2-FPR))/SQRT(1.-FFF2))   C  016:001712
      GOTO20                                                            C  016:001F10
   10 A=HEM-2.*HE+2.*HE*FFE*ALOG((FFE+1.)/(FFE-1.))                     C  016:001F13
      A=A+HFM-2.*HF-HEM-2.*HE+HF*FFF*ALOG((1.+FFF)/(1.-FFF))            C  016:002514
   20 A=A+2.*SQRT((1./X**2)-1.)                                         C  016:002D15
      RETURN                                                            C  016:003112
      END                                                               C  016:003115
                                                                   SEGMENT 016 IS 003D LONG
```

```
                                                                    START OF SEGMENT 017
      SUBROUTINE DELAYE(R,FPE,F,X,T,D1T,D2T,D3T)                   C 017:000010
      COMMON HE,HEM,HF,HFM,TIME                                    C 017:000010
                                                                   C 017:000010
C          EVALUATES DELAY AND DELAY DERIVATIVES FOR E REGION      C 017:000010
C              HE  = THICKNESS OF E REGION (KM.)                   C 017:000010
C              HEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)  C 017:000010
C              FPE = PENETRATION FREQUENCY OF E REGION (MHZ.)      C 017:000010
C              R   = LENGTH OF GROUND PATH (KM.)                   C 017:000010
C              F   = FREQUENCY (MHZ.)                              C 017:000010
C              X   = COS(THETA); THETA=RAY ANGLE MEASURED FROM VERTICAL  C 017:000010
C              T   = SIGNAL PATH DELAY (SEC.)                      C 017:000010
C              D1T = DERIVATIVE OF DELAY (SEC./MHZ.)               C 017:000010
C              D2T = SECOND DERIVATIVE OF DELAY (SEC./MHZ.**2)     C 017:000010
C              D3T = THIRD DERIVATIVE OF DELAY (SEC./MHZ.**3)      C 017:000010
                                                                   C 017:000010
      C=3.E+05                                                     C 017:000010
      FNE=F/FPE                                                    C 017:000213
      FNE2=FNE**2                                                  C 017:000315
      FNE4=FNE**4                                                  C 017:000414
      FFE=FNE*X                                                    C 017:000611
      FFE2=FFE**2                                                  C 017:000713
      X2=X**2                                                      C 017:000812
      X4=X**4                                                      C 017:000913
      V1=X2/(1.-X2)                                                C 017:000B10
      V2=V1**.5                                                    C 017:000C14
      V3=V1**1.5                                                   C 017:000F13
      V4=1./((1.-X2)**1.5)                                         C 017:001213
      V5=1./((1.-X2)**2.5)                                         C 017:001614
      V6=X2*V5                                                     C 017:001A14
      W1=1./(1.-FFE2)                                              C 017:001C10
      W2=W1**2                                                     C 017:001D13
      W3=W1**3                                                     C 017:001E12
      Z1=X/F                                                       C 017:001F15
      T=R/(C*SQRT(1.-X2))                                          C 017:002111
      T2=T**2                                                      C 017:002410
      U1=X*T/(1.-X2)                                               C 017:002415
      U2=(1.+2.*X2)/(X2*T)                                         C 017:002710
      DEN=R*V3+2.*HEM-4.*HE*W1                                     C 017:002914
      DXDF=Z1*(R*V2-2.*HEM+4.*HE*W1)/DEN                           C 017:002D15
      DXDF2=DXDF**2                                                C 017:003310
      D1T=U1*DXDF                                                  C 017:003315
      Z2=Z1+DXDF                                                   C 017:003511
      Z3=Z2**2                                                     C 017:003613
      Z4=1.-3.*X*F*DXDF/(1.-X2)                                    C 017:003712
      D2XDF=(Z1*(R*V4*Z4*DXDF+8.*HE*F*FNE2*W2*Z3)/DEN)             C 017:003A15
      D2XDF=D2XDF+(DXDF2/X)-(DXDF/F)                               C 017:004112
      D2T=U2*(D1T**2)+U1*D2XDF                                     C 017:004315
      Z5=D2XDF-(DXDF2/X)+(DXDF/F)                                  C 017:004613
      Z6=3.*X*DXDF2+(1.-X2)*D2XDF                                  C 017:004912
      Z7=DXDF2*((1.+X2)/(1.-X2))+Z1*DXDF+X*D2XDF                   C 017:004C13
      Z8=Z1+5.*DXDF+2.*F*D2XDF                                     C 017:005110
      D3XDF=-((DXDF**3)/X2)+(2.*DXDF*D2XDF/X)+(DXDF/(F**2))-(D2XDF/F)-Z5  C 017:005412
     1*((Z1*DXDF)/X)-(Z5*(R*DXDF*V6+8.*HE*X*FNE2*Z2*W2)/DEN)       C 017:005A12
      D3XDF=D3XDF+(Z1/DEN)*(R*V5*Z4*Z6-3.*R*F*DXDF*V5*Z7+32.*HE*X*F*FNE4  C 017:006213
     1*W3*(Z2**3)+8.*HE*FNE2*Z2*Z8*W2)                             C 017:006A11
      D3T=-(3.*(1.+X2+2.*X4)*(D1T**3)/(X4*T2))+3.*U2*D1T*D2T+U1*D3XDF  C 017:007013
      RETURN                                                       C 017:007613
      END                                                          C 017:007910
                                                                   SEGMENT 017 IS 008E LONG
```

```
                                                                              START OF SEGMENT 018
      SUBROUTINE DELAYF(R,FPE,FPF,F,X,T,D1T,D2T,D3T)                          C  0181000010
      COMMON HE,HEM,HF,HFM,TIME                                               C  0181000010
                                                                              C  0181000010
C           EVALUATES DELAY AND DELAY DERIVATIVES FOR F REGION                C  0181000010
C              HE = THICKNESS OF E REGION (KM.)                               C  0181000010
C              HEM = HEIGHT OF MAX. ELECTRON DENSITY IN E REGION (KM.)        C  0181000010
C              FPE = PENETRATION FREQUENCY OF E REGION (MHZ.)                 C  0181000010
C              HF = THICKNESS OF F REGION (KM.)                               C  0181000010
C              HFM = HEIGHT OF MAX. ELECTRON DENSITY IN F REGION (KM.)        C  0181000010
C              FPF = PENETRATION FREQUENCY OF F REGION (MHZ.)                 C  0181000010
C              R   = LENGTH OF GROUND PATH (KM.)                              C  0181000010
C              F   = FREQUENCY (MHZ.)                                         C  0181000010
C              T   = SIGNAL PATH DELAY (SEC.)                                 C  0181000010
C              D1T = DERIVATIVE OF DELAY (SEC./MHZ.)                          C  0181000010
C              D2T = SECOND DERIVATIVE OF DELAY (SEC./MHZ.**2)                C  0181000010
C              D3T = THIRD DERIVATIVE OF DELAY (SEC./MHZ.**3)                 C  0181000010
                                                                              C  0181000010
      TN='NITE'                                                               C  0181000010
      C=3.E+05                                                                C  0181000213
      FNE=F/FPE                                                               C  0181000413
      FNE2=FNE**2                                                             C  0181000515
      FFE=FNE*X                                                               C  0181000614
      FFE2=FFE**2                                                             C  0181000610
      FNF=F/FPF                                                               C  0181000815
      FNF2=FNF**2                                                             C  0181000A11
      FFF=FNF*X                                                               C  0181000B10
      FFF2=FFF**2                                                             C  0181000C12
      FFF3=FFF**3                                                             C  0181000D11
      FFF4=FFF**4                                                             C  0181000E14
      FFF5=FFF4*FNF                                                           C  0181001011
      FFF6=FFF2*FNF                                                           C  0181001111
      FPR=FPE/FPF                                                             C  0181001213
      FPR2=FPR**2                                                             C  0181001315
      RFPR=SQRT(1.-FPR2)                                                      C  0181001414
      X2=X**2                                                                 C  0181001613
      X4=X**4                                                                 C  0181001714
      V1=1./SQRT(1.-X2)                                                       C  0181001911
      V2=X*V1                                                                 C  0181001B12
      V3=X*X2*(V1**3)                                                         C  0181001C14
      V4=V1**3                                                                C  0181001F11
      W1=1./(FFE2-1.)                                                         C  0181002014
      W2=W1**2                                                                C  0181002211
      W3=W1**3                                                                C  0181002310
      W4=(FFE2/(FFF2-1.))**1.5                                                C  0181002413
      W5=W1**2.5                                                              C  0181002613
      W6=W1*W5                                                                C  0181002B13
      W7=1./(1.-FFF2)                                                         C  0181002C15
      W8=W7**2                                                                C  0181002E12
      W9=W7**3                                                                C  0181002F11
      W10=FFF2-FPR2                                                           C  0181003014
      W11=SQRT(W10)                                                           C  0181003210
      T=R*V1/C                                                                C  0181003313
      T2=T**2                                                                 C  0181003512
      Z1=X/F                                                                  C  0181003611
      IF(TIME.IS.TN)GOTO10                                                    C  0181003713
      C1=2.*(HFM-HEM-2.*HF*RFPR)                                              C  0181003911
      C2=RFPR*W11+W10                                                         C  0181003B12
      B=2.*HEM+4.*HE*W1+C1*W4-(4.*HF*FFF3/C2)-4.*HF*FFF3*W7                   C  0181003F11
      DXDF=Z1*(R*V2-B)/(R*V3+B)                                               C  0181004B10
```

```
      GOTO20                                                              C  0181004B15
10    R=2.*HF*W+8.*HE*W1-4.*HF*W7                                         C  0181004C12
      DXDF=Z1*(R*V2-B)/(R*V3+B)                                           C  0181005113
90    D1T=X*T*DXDF/(1.-X2)                                                C  0181005512
      Z2=Z1+DXDF                                                          C  0181005810
      DEN=R*V3+B                                                          C  0181005912
      IF(TIME.IS.TN)GOTO30                                                C  0181005B11
      ZD1=2.+(RFPR/W11)                                                   C  0181005C15
      C3=C2**2                                                            C  0181005E14
      FFE6=FFE2*FNF                                                       C  0181005F15
      D8DF=Z2*(8.*HE*X*FNE2*W2-3.*C1*FFE6*W5-(12.*HF*FFF6/C2)+(4.*HF*FFF  C  0181006111
     15*ZD1/C3)-12.*HF*FFF6*W7-8.*HF*FFF5*W8)                             C  0181006913
      D2XDF=(DXDF**2/X)-(DXDF/F)+((R*Z1*DXDF*V4*(1.-(3.*X*F*DXDF/(1.-X2   C  0181007013
     1)))-Z2*D8DF)/DEN)                                                   C  0181007614
      GOTO40                                                              C  0181007915
30    D2XDF=(DXDF**2/X)-(DXDF/F)+Z1*((R*DXDF*V4*(1.-(3.*X*F*DXDF/(1.-X2   C  0181007A12
     1)))+16.*HE*F*FNE2*W2*(Z2**2)+8.*HF*F*FNF2*W8*(Z2**2))/DEN)          C  0181008012
40    D2T=((D1T**2)*(1.+2.*X2)/(X2*T))+(D2XDF*X*T/(1.-X2))                C  0181008915
      IF(TIME.IS.TN)GOTO50                                                C  0181008F14
      C4=C2**3                                                            C  0181009112
      ZD2=ZD1**2                                                          C  0181009215
      FFF7=FFF3*FFF2*FNF2                                                 C  0181009410
      FFF8=FFF*FNF2                                                       C  0181009515
      FFF9=FFF3*FNF2                                                      C  0181009711
      W12=1./(W11**3)                                                     C  0181009813
      ZD3=2.*Z1+3.*DXDF                                                   C  0181009A12
      ZD4=3.*Z1+2.*DXDF                                                   C  0181009C14
      ZD5=2.*Z1+DXDF                                                      C  0181009F10
      ZD6=5.*Z1+4.*DXDF                                                   C  018100A015
      ZD7=5.-FFF2                                                         C  018100A311
      ZD8=3.-FFF2                                                         C  018100A413
      ZD9=D2XDF                                                           C  018100A515
      ZD10=3.*ZD4-FFF2*ZD6                                                C  018100A614
      D2BDF=((DXDF-Z1*F*D2XDF)/(F*Z2))*D8DF+Z2*(-8.*HE*FNE2*(FFE2*ZD3+Z   C  018100A910
     1D5)*W3+FFF8*3.*C1*(ZD4+FFE2*ZD3)*W6+(12.*HF*FFF6*(-ZD4+FFF2*Z2*ZD   C  018100AF11
     21)/C2)+(4.*HF*FFF9*ZD6*ZD1/C3)-(4.*HF*FFF7*RFPR*Z2*W12/C3)-(8.*HF   C  018100B015
     3*FFF7*Z2*ZD2/C4)-16.*HF*FFF9*Z2*ZD8*W9-4.*HF*FFF8*ZD10*W8)          C  018100BF15
      DXDF2=DXDF**2                                                       C  018100C915
      D3XDF=-((DXDF**3)/X2)+(2.*DXDF*D2XDF/X)+(DXDF/(F**2))-(D2XDF/F)+(   C  018100CB10
     11./DEN)*(-3.*R*X*DXDF*(V1**5)*(((1.+4.*X2)/(1.-X2))*DXDF2+Z1*DXDF   C  018100D110
     2+3.*X*D2XDF)+(R*V4/F)*(((1.+2.*X2)/(1.-X2))*DXDF2-Z1*DXDF+X*ZD9)-   C  018100D810
     3(1./F)*(2.*DXDF*Z1+2.*F*D2XDF-(DXDF2/Z1))*D8DF-Z2*D2BDF)            C  018100E013
      GOTO60                                                              C  018100E714
50    DXDF2=DXDF**2                                                       C  018100E811
      ZN1=D2XDF-(DXDF2/X)+(DXDF/F)                                        C  018100E912
      ZN2=Z1+3.*DXDF+2.*F*D2XDF                                           C  018100EC11
      ZN3=V1**5                                                           C  018100EF13
      ZN4=Z2**3                                                           C  018100F112
      ZZ=Z1-DXDF                                                          C  018100F215
      D3XDF=-((DXDF**3)/X2)+(2.*DXDF*D2XDF/X)+(DXDF/(F**2))-(D2XDF/F)-(   C  018100F411
     1ZZ*ZN1/X)-(ZN1/DEN)*(R*3.*X2*DXDF*ZN3-16.*HF*X*FNE2*Z2*W2-8.*HF*X   C  018100FA11
     2*FNF2*ZZ*W8)+(Z1/DEN)*(R*ZN3*(1.-(3.*X*F*DXDF/(1.-X2)))*(3.*X*DXD   C  0181010311
     3F2+(1.-X2)*D2XDF)-3.*R*F*DXDF*ZN3*(((1.+X2)/(1.-X2))*DXDF2+Z1*DXD   C  0181010B12
     4F+X*D2XDF)-64.*HE*X*F*(FNE2**2)*ZN4*W3+32.*HF*X*F*(FNF2**2)*ZN4*W   C  0181011210
     59+16.*HE*FNE2*Z2*ZN2*W2+8.*HF*FNF2*Z2*ZN2*W8)                       C  0181011B13
80    D3T=-(3./(X4*T2))*(1.+X2+2.*X4)*(D1T**3)+(3./(X2*T))*(1.+2.*X2)*D   C  0181012313
     11T*D2T+(X*T/(1.-X2))*D3XDF                                          C  0181012A14
      RETURN                                                              C  0181012E14
      END                                                                 C  0181012F11
                                                          SEGMENT 018 IS 014B LONG
```

APPENDIX C
COMPUTER GENERATION OF RANDOM GAINS

APPENDIX C
COMPUTER GENERATION OF RANDOM GAINS

For each ionosphere return, it is desired to generate a stationary, zero-mean Gaussian sequence $\tilde{G}_j(nT_o)$, $n = 0, 1, \cdots$, having the correlation function

$$C_{\tilde{G}_j}(nT_o) = E\, \tilde{G}_j(nT_o + mT_o)\, \tilde{G}_j^*(mT_o)$$

$$= A_j^{\,2}\left[1 + (2\pi\rho\sigma_j T_o|n|) + \frac{1}{3}\,(2\pi\rho\sigma_j T_o|n|)^2\right]$$

$$\cdot \exp\left[-2\pi\rho\sigma_j T_o|n| + 2\pi i\nu_j T_o n\right] \qquad \text{(C.1)}$$

$$n = 0, 1, \cdots$$
$$j = 1, \cdots, N\ .$$

We will generate this sequence by passing a white noise process through an appropriately chosen discrete-time linear filter [26]. Denoting the filter input by $\{\xi_j(n)\}$ and the impulse response by $\{a_j(n)\}$ we have

$$\tilde{G}_j(nT_o) = \exp[2\pi i\nu_j nT_o] \sum_{m=0}^{\infty} \xi_j(n-m)\, a_j(m) \qquad \text{(C.2)}$$

$$j=1, \cdots N$$

where

$$a_j(m) = \begin{cases} A_j\left(\frac{8}{3}\pi\rho\sigma_j T_o\right)^{\frac{1}{2}} (2\pi\rho\sigma_j mT_o)^2 \exp\left[-2\pi\rho\sigma_j mT_o\right], & m \geq 0 \\ 0\ , & m<0 \end{cases} \qquad , \quad j=1, \cdots N. \qquad \text{(C.3)}$$

The random variables $\{\xi_j(n)\}$ are independent, identically-distributed, complex Gaussian variables. They may be expressed as

$$\xi_j(n) = \xi_j^{(1)}(n) + i\xi_j^{(2)}(n) \quad , \quad n=0,1,\cdots \tag{C.4}$$

where $\xi_j^{(1)}(n)$ and $\xi_j^{(2)}(n)$ are, respectively, the real and imaginary parts of $\xi_j(n)$. For each n the variables $\xi_j^{(1)}(n)$ and $\xi_j^{(2)}(n)$ are independent, zero-mean, unit variance, real Gaussian variables, and both components of $\xi_j(n)$ are independent of each other for all values of n.

To generate $\tilde{G}_j(nT_o)$ directly using the convolution (C.2), we would have to approximate the infinite summation with a sufficiently accurate truncated summation. Because $\sigma_j T_o$ is very small, the length of this truncated summation would be large and we would expect error accumulation and an attendant loss of accuracy. However, our particular choice of an approximating correlation function (C.1) allows us to generate the random sequence $G_j(nT_o)$ iteratively. This was, of course, the primary consideration in selecting this particular approximation. The iterative scheme is given below

$$\begin{aligned}
\tilde{G}_j(nT_o + 3T_o) = {} & K_j \exp[-2\pi\rho\sigma_j T_o + 2\pi i\nu_j(n+3)T_o]\ \xi_j(n+2) \\
& + K_j \exp[-4\pi\rho\sigma_j T_o + 2\pi i\nu_j(n+3)T_o]\ \xi_j(n+1) \\
& + \exp[-6\pi\rho\sigma_j T_o + 6\pi i\nu_j T_o]\ \tilde{G}_j(nT_o) \\
& - 3\exp[-4\pi\rho\sigma_j T_o + 4\pi i\nu_j T_o]\ \tilde{G}_j(nT_o + T_o) \\
& + 3\exp[-2\pi\rho\sigma_j T_o + 2\pi i\nu_j T_o]\ \tilde{G}_j(nT_o + 2T_o)
\end{aligned} \tag{C.5}$$

$$n \geq 0$$
$$j=1,\cdots N$$

where

$$K_j = \frac{2}{\sqrt{3}} \; A_j \; (2\pi\rho\sigma_j T_o)^{5/2} \quad , \quad j=1,\cdots N \tag{C.6}$$

and, as discussed earlier, we will choose $\rho = 2.1$. To start this iteration we need $\tilde{G}_j(0)$, $\tilde{G}_j(T_o)$ and $\tilde{G}_j(2T_o)$ given by

$$\tilde{G}_j(0) = A_j W_j(1) \tag{C.7}$$

$$\tilde{G}_j(T_o) = \exp[2\pi i \nu_j T_o] \cdot \left[\gamma_j(1) W_j(2) + \lambda_j(1)\, \tilde{G}_j(0)\right] \tag{C.8}$$

$$\tilde{G}_j(2T_o) = \exp[4\pi i \nu_j T_o] \cdot \Big\{\gamma_j(2) W_j(3) + K_j \exp[-2\pi\rho\sigma_j T_o]\, \xi_j(1)$$

$$+ \lambda_j(1)[1-\lambda_j(2)]\, \tilde{G}_j(T_o) + \lambda_j(2)\, \tilde{G}_j(0)\Big\} \tag{C.9}$$

$$j=1,\cdots N.$$

Here $W_j(1)$, $W_j(2)$, $W_j(3)$, and $\xi_j(1)$ are independent, complex, Gaussian variables whose real and imaginary parts are independent and have zero mean and unit variance. The other terms appearing in (C.7)-(C.9) are

$$\lambda_j(1) = [1+(4\pi\rho\sigma_j T_o) + \tfrac{1}{3}(4\pi\rho\sigma_j T_o)^2] \cdot \exp[-4\pi\rho\sigma_j T_o] \tag{C.10}$$

$$\lambda_j(2) = [1+(8\pi\rho\sigma_j T_o) + \tfrac{1}{3}(8\pi\rho\sigma_j T_o)^2] \cdot \exp[-8\pi\rho\sigma_j T_o] \tag{C.11}$$

$$\gamma_j(1) = A_j \, [1-\lambda^2{}_j(1)]^{1/2} \tag{C.12}$$

$$\gamma_j(2) = A_j \left[\left(1-\lambda^2{}_j(1)\right)\left(1-\lambda^2{}_j(2)\right) - \frac{4}{3}\left(2\pi\rho\sigma_j T_o\right)^5 \exp[-4\pi\rho\sigma_j T_o]\right]^{1/2} \tag{C. 13}$$

The coefficients A_j are determined by the attenuation of each return via Eqn. (43).

It remains to generate the sequence of independent, Gaussian variables $\{\xi_j(n)\}$. We begin by using the congruence method to generate a sequence of uniform random variables. This method generates a sequence of random integers via the iterative scheme [24]

$$Y_{n+1} = a \cdot Y_n \ [\mathrm{mod}\ M] \ . \tag{C.14}$$

For the quantities a and M, we have found $a = 7^5$ and $M = 2^{31}$ to be effective. Multiplication is done using double precision arithmetic but the final result is expressed as a single precision variable. Upon division of $\{Y_n\}$ by 2^{31} a sequence of independent, uniform [0,1] variables is produced. This sequence is then transformed into a sequence of independent, complex, Gaussian variables. If u_1 and u_2 are real, independent, uniform variables, the real and imaginary parts of the variable ξ defined by

$$\xi = (-2 \ln u_1)^{\frac{1}{2}} \exp[2\pi i u_2] \tag{C.15}$$

are independent and Gaussian with zero mean and unit variance [24]. Thus, we obtain the desired sequence of complex Gaussian variables for use as the filter input in Eqn. (C.5).

APPENDIX D
SIGNAL ANALYSIS PROGRAM

```
 B6700/B7700   F O R T R A N   C O M P I L A T I O N   M A R K   2.8.060    MONDAY, 09/26/77    02:22 PM

FILE  2(KIND=TAPE,TITLE="HFOUTPUT.")                                         C  00000001
FILE  10(KIND=TAPE,TITLE="HFINPUT.")                                         C  00000002
                                                                              START OF SEGMENT 002
                                                                            FORMAT SEGMENT IS 00CY LONG
                                                                            FORMAT SEGMENT IS 00EA LONG
      DIMENSION T(10,14),D2(10),D3(10),N1(10)                                C  002:000010
      COMPLEX X(10,4),G(90),Y(10),R(10),D(10,801),B(10),SIGIN(5000),AI       C  002:000010
      COMPLEX PRTOUT(10000),GNDOUT,SIGOUT                                    C  002:000010
      DOUBLE PRECISION C(10,7)                                               C  002:000010
                                                                             C  002:000010
C          THIS PROGRAM IS THE SIGNAL ANALYSIS PART OF THE SIMULATOR         C  002:000010
                                                                             C  002:000010
C              FC     = CARRIER FREQUENCY (MHZ.)                             C  002:000010
C              BW     = BANDWIDTH (KHZ.)                                     C  002:000010
C              RANGE  = DISTANCE BETWEEN TRANSMITTER AND RECEIVER (KM.)      C  002:000010
C              TIME   = TIME OF DAY ('DAY' OR 'NITE')                        C  002:000010
C              NPATH  = NUMBER OF RETAINED IONOSPHERE RETURNS                C  002:000010
C              NUMIN  = NUMBER OF INPUT SAMPLES                              C  002:000010
C              NTAPE  = TAPE WRITE SELECTOR (WRITES TAPE IF NTAPE=1).        C  002:000010
C              NGAIN  = RANDOM GAIN SELECTOR (MULTIPLIES BY RANDOM GAINS     C  002:000010
C                       ONLY IF NGAIN=1)                                     C  002:000010
C              NPRINT= NUMBER OF OUTPUT VALUES TO BE PRINTED                 C  002:000010
C              NCONV = ONE-SIDED LENGTH OF TRUNCATED IMPULSE RESPONSE        C  002:000010
C              GDELAY= GROUNDWAVE DELAY (SEC.)                               C  002:000010
C              GATTEN= GROUNDWAVE ATTENUATION (DB)                           C  002:000010
C              ALPHA  = PARAMETER USED IN APPROXIMATE SPECTRUM               C  002:000010
C              NGND   = GROUNDWAVE DELAY SHIFT PARAMETER                     C  002:000010
C              ISEED  = SEED VALUE USED IN RANDOM NUMBER GENERATOR. IT       C  002:000010
C                       IS SET = 7**5 FOR FIRST CALL AND IT IS REPLACED      C  002:000010
C                       BY A NEW VALUE TO BE USED IN SUBSEQUENT CALLS        C  002:000010
C              N1MIN  = MINIMUM DELAY SHIFT PARAMETER                        C  002:000010
C              N1MAX  = MAXIMUM DELAY SHIFT PARAMETER                        C  002:000010
C              GNDOUT= DELAYED, ATTENUATED GROUNDWAVE                        C  002:000010
C              SIGOUT= OUTPUT SIGNAL (GROUNDWAVE+IONOSPHERE RETURNS)         C  002:000010
                                                                             C  002:000010
C          THE ARRAY D(I,N) CONTAINS TERMS IN IMPULSE RESPONSE. IT IS        C  002:000010
C          PRODUCED BY THE SUBROUTINE 'IMPULS'.                              C  002:000010
                                                                             C  002:000010
C          THE ARRAY D2(I) CONTAINS AMPLITUDE DISTORTION COEFFICIENTS        C  002:000010
C          THE ARRAY D3(I) CONTAINS PHASE DISTORTION COEFFICIENTS            C  002:000010
C          THE ARRAY N1(I) CONTAINS SIGNAL DELAY SHIFT PARAMETERS            C  002:000010
C          THE ARRAY SIGIN(N) CONTAINS INPUT SIGNAL SEQUENCE                 C  002:000010
                                                                             C  002:000010
C          THE ARRAY G(I) CONTAINS THE RANDOM GAINS. IT IS PRODUCED          C  002:000010
C          BY THE SUBROUTINE 'GAIN'.                                         C  002:000010
                                                                             C  002:000010
C          THE ARRAYS C(I,J), X(I,J) AND Y(I) CONTAIN VARIABLES USED IN      C  002:000010
C          SUBROUTINE 'GAIN'. THEY ARE SET = 0 FOR FIRST CALL.               C  002:000010
                                                                             C  002:000010
C          THE ARRAY T(I,J) CONTAINS THE IONOSPHERE CHANNEL PARAMETERS.      C  002:000010
C          ITS ENTRIES ARE:                                                  C  002:000010
C              I =       INDEX NUMBER FOR PATH                               C  002:000010
C              J = 1:  NUMBER OF HOPS                                        C  002:000010
C                = 2:  REGION AND MAGNETOIONIC INDICATOR (EO = E REGION -    C  002:000010
C                        ORDINARY WAVE; SIMILARLY FOR FX, FO, AND FX)        C  002:000010
C                = 3:  'HIGH' (FOR HIGH RAY) OR 'LOW' (FOR LOW RAY)          C  002:000010
C                = 4:  SOLUTION INDICATOR (0=NO SOLUTION, 1=SOLUTION)        C  002:000010
C                = 5:  THETA = RAY ANGLE MEASURED FROM VERTICAL (DEGREES)    C  002:000010
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C                = 6: PATH LENGTH (KM.)                                   C   002:000010
C                = 7: DELAY AT CARRIER FREQUENCY (SEC.)                   C   002:000010
C                = 8: RECEIVED CARRIER PHASE (CYCLES)                     C   002:000010
C                = 9: SIGNAL DELAY (SEC.)                                 C   002:000010
C                =10: AMPLITUDE DISTORTION (SEC/MHZ)                      C   002:000010
C                =11: PHASE DISTORTION (SEC/MHZ**2)                       C   002:000010
C                =12: ATTENUATION (DB.)                                   C   002:000010
C                =13: DOPPLER SHIFT (HZ.)                                 C   002:000010
C                =14: DOPPLER SPREAD (HZ.)                                C   002:000010
C          WARNING: NPATH MUST NOT EXCEED 10                              C   002:000010
C          WARNING: NCONVL MUST NOT EXCEED 400 (HOWEVER SEE WARNING IN    C   002:000010
C                   SUBROUTINE 'IMPULS').                                 C   002:000010
C          WARNING: NPRINT MUST NOT EXCEED 10,000                         C   002:000010
                                                                          C   002:000010
      AI=(0.,1.)                                                          C   002:000010
      PI=3.14159265359                                                    C   002:000111
      ALPHA=2.1                                                           C   002:000610
      INITF='WITE'                                                        C   002:000813
C          INITIALIZE VARIABLES AND ARRAYS FOR SUBROUTINE 'GAIN'          C   002:000A13
      ISEED=7**5                                                          C   002:000A13
      NCALL=0                                                             C   002:000C13
      DO10I=1,10                                                          C   002:000D11
      Y(I)=(0.,0.)                                                        C   002:000E10
      DO4J=1,7                                                            C   002:001010
      C(I,J)=DBLE(0.)                                                     C   002:001110
    4 CONTINUE                                                            C   002:001315
      DO6J=1,4                                                            C   002:001610
      X(I,J)=(0.,0.)                                                      C   002:001710
    6 CONTINUE                                                            C   002:001A11
   10 CONTINUE                                                            C   002:001C12
C          END INITIALIZATION FOR SUBROUTINE 'GAIN'                       C   002:001E13
C          READ INPUT DATA                                                C   002:001E13
      READ/,FC,BW,RANGE,NPATH                                             C   002:001E13
                                                                              FIB IS 0006 LONG
      READ(5,900)TIME                                                     C   002:002610
      READ/,GDELAY,GATTEN                                                 C   002:003312
      READ/,NCONVL                                                        C   002:003D10
      READ/,NUMIN                                                         C   002:004510
      READ/,NGAIN                                                         C   002:004D10
      READ/,NPRINT                                                        C   002:005510
      READ/,NTAPE                                                         C   002:005D10
      DO20I=1,NPATH                                                       C   002:006510
      READ(5,955)(T(I,J),J=1,7)                                           C   002:006610
      READ(5,956)(T(I,J),J=8,11)                                          C   002:007412
      READ(5,957)(T(I,J),J=12,14)                                         C   002:008212
   20 CONTINUE                                                            C   002:009012
C          END READ INPUT DATA                                            C   002:009213
C          DETERMINE IF LIMITS FOR NPATH, NCONVL OR NPRINT ARE EXCEEDED   C   002:009213
      IF(NPATH-10)22,22,21                                                C   002:009213
   21 WRITE(6,901)                                                        C   002:009315
                                                                              FIB IS 0006 LONG
      GOTO999                                                             C   002:009812
   22 IF(NCONVL-400)26,26,24                                              C   002:009B15
   24 WRITE(6,902)                                                        C   002:009A12
   26 IF(NPRINT-10000)28,28,27                                            C   002:009E12
   27 WRITE(6,905)                                                        C   002:009F15
      GOTO999                                                             C   002:00A412
C          END DETERMINATION OF EXCEEDED LIMITS                           C   002:00A415
C          THE SAMPLE INTERVAL TO IS HALF THE NYQUIST SAMPLE INTERVAL     C   002:00A415
   28 TO=1./(BW*2000.)                                                    C   002:00A415
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C           INITIALIZE ARRAYS FOR CONVOLUTION                            C  002:00A6:14
      DO40I=1,NPATH                                                      C  002:00A6:14
      D2(I)=T(I,10)*2.*PI*(BW*.2)                                        C  002:00A6:10
      D3(I)=T(I,11)*2.*PI*(BW*.3)*(1.E-03)                               C  002:00AC:12
      NN1=ABS(T(I,9))/TO                                                 C  002:00B2:12
      XN1=TO-TO*FLOAT(NN1)                                               C  002:00B5:10
      IF(T(I,9))32,34,34                                                 C  002:00B6:14
   32 NN2=-1                                                             C  002:00B6:14
      GOTO36                                                             C  002:00B9:13
   34 NN2=1                                                              C  002:00BA:10
   36 IF(XN1-.5)38,38,39                                                 C  002:00BA:14
   38 N1(I)=NN1*NN2                                                      C  002:00BD:14
      GOTO40                                                             C  002:00C0:10
   39 N1(I)=(NN1+1)*NN2                                                  C  002:00C0:13
   40 CONTINUE                                                           C  002:00C3:11
C           END INITIALIZATION OF ARRAYS FOR CONVOLUTION                 C  002:00C5:12
C           DETERMINE NGND                                               C  002:00C5:12
      NG1=GDELAY/TO                                                      C  002:00C5:12
      XG1=TO-TO*FLOAT(NG1)                                               C  002:00C6:15
      IF(XG1-.5)42,42,44                                                 C  002:00C8:13
   42 NGND=NG1                                                           C  002:00CA:14
      GOTO45                                                             C  002:00CB:13
   44 NGND=NG1+1                                                         C  002:00CC:10
C           END DETERMINATION OF NGND                                    C  002:00CD:12
C           DETERMINE MAX. AND MIN. DELAY SHIFT PARAMETERS               C  002:00CD:12
   45 LLIMIT=2*NCONVL+1                                                  C  002:00CE:12
      N1MAX=NGND                                                         C  002:00CF:11
      N1MIN=NGND                                                         C  002:00D0:10
      DO48I=1,NPATH                                                      C  002:00D0:15
      N1MAX=MAX0(N1MAX,N1(I))                                            C  002:00D2:10
      N1MIN=MIN0(N1MIN,N1(I))                                            C  002:00D5:14
   48 CONTINUE                                                           C  002:00D9:12
C           END DETERMINATION OF MAX. AND MIN. SHIFT PARAMETERS          C  002:00D8:13
C           READ INITIAL INPUT SEQUENCE                                  C  002:00D8:13
      NREAD=N1MAX-N1MIN+LLIMIT                                           C  002:00DB:13
      NREAD1=NREAD-1                                                     C  002:00D0:13
      IF(NREAD-5000)50,50,55                                             C  002:00DL:13
   55 WRITE(6,903)                                                       C  002:00E0:10
      GOTO999                                                            C  002:00E4:12
   50 DO60L=1,NREAD                                                      C  002:00E4:15
      XLL=FLOAT(L-1)                                                     C  002:00E6:10
      READ(10,904)SIGIN(L)                                               C  002:00E7:11
   60 CONTINUE                                                           C  002:00F2:12
C           END READ INITIAL INPUT SEQUENCE                              C  002:00F4:13
      NSHIFT=0                                                           C  002:00F4:13
      CALL IMPULS(NCONVL,NPATH,D2,D3,D)                                  C  002:00F5:11
                                                                         C  002:00F6:15
C           DO 200 IS THE FUNDAMENTAL LOOP FOR DETERMINING THE OUTPUT    C  002:00F8:15
C           FOR EACH INDEX N (N CORRESPONDS TO AN ARGUMENT OF (N-1)*TO). C  002:00F8:15
                                                                         C  002:00F8:15
      DO200N=1,NUMIN                                                     C  002:00F8:15
C           DETERMINE DISTORTED IONOSPHERE RETURNS                       C  002:00FA:10
      DO70I=1,NPATH                                                      C  002:00FA:10
      B(I)=(0.,0.)                                                       C  002:00FB:10
   70 CONTINUE                                                           C  002:00FD:10
      DO100L=1,LLIMIT                                                    C  002:00FF:11
      LU=2*NCONVL-L+2                                                    C  002:0100:10
      LL=L-NCONVL+N*NSHIFT-1                                             C  002:0102:13
      DO80I=1,NPATH                                                      C  002:0105:12
      LS=LL-N1(I)                                                        C  002:0106:10
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      IF(LS)80,80,75                                              C  0021010611
75    B(I)=B(I)+D(I,LD)*SIGIN(IS)                                 C  0021010910
80    CONTINUE                                                    C  0021011210
100   CONTINUE                                                    C  0021011411
      DO110I=1,NPATH                                              C  0021011612
      B(I)=B(I)*CEXP(-AI*2.*PI*T(I,8))                            C  0021011710
110   CONTINUE                                                    C  0021012213
C           END DETERMINATION OF DISTORTED IONOSPHERE RETURNS     C  0021012414
C           MULTIPLY BY RANDOM GAINS IF NGAIN=1                   C  0021012414
      IF(NGAIN-1)120,115,120                                      C  0021012414
115   CALL GAIN(ISFED,NCALL,BW,ALPHA,NPATH,T,C,X,Y,G)             C  0021012515
      DO120I=1,NPATH                                              C  0021012612
      B(I)=B(I)*G(I)                                              C  0021012010
120   CONTINUE                                                    C  0021013114
C           END MULTIPLY BY RANDOM GAINS                          C  0021013315
C           DETERMINE DELAYED, ATTENUATED GROUNDWAVE              C  0021013315
      GNDOUT=(0.,0.)                                              C  0021013315
      LG=N-NGND-NSHIFT                                            C  0021013510
      IF(LG)125,125,122                                           C  0021013710
122   GNDOUT=SIGIN(LG)*(10.**(-GATTEN/20.))                       C  0021013715
125   CONTINUE                                                    C  0021013014
C           END DETERMINATION OF DELAYED, ATTENUATED GROUNDWAVE   C  0021013014
C           ADD GROUNDWAVE AND IONOSPHERE RETURNS                 C  0021013014
      SIGOUT=GNDOUT                                               C  0021013014
      DO130I=1,NPATH                                              C  0021013813
      SIGOUT=SIGOUT+B(I)                                          C  0021014010
130   CONTINUE                                                    C  0021014410
C           END ADDITION OF GROUNDWAVE AND IONOSPHERE RETURNS     C  0021014611
C           WRITE TAPE (FILE 2) IF NTAPE = 1                      C  0021014611
      IF(NTAPE-1)134,132,134                                      C  0021014611
132   WRITE(2,904)SIGOUT                                          C  0021014712
134   CONTINUE                                                    C  0021015012
C           END WRITE TAPE                                        C  0021015012
      IF(NPRINT-N)136,135,135                                     C  0021015012
135   PRTOUT(N)=SIGOUT                                            C  0021015114
136   CONTINUE                                                    C  0021015312
      IF(N-N1MAX-NCONVL)200,200,140                               C  0021015312
140   NSHIFT=NSHIFT+1                                             C  0021015511
      DO150L=1,NREAD1                                             C  0021015613
      LI=L+1                                                      C  0021015610
      SIGIN(L)=SIGIN(LI)                                          C  0021015912
150   CONTINUE                                                    C  0021015814
      IF(NREAD+NSHIFT-NUMIN)154,154,156                           C  0021015015
154   READ(10,904)SIGIN(NREAD)                                    C  0021015714
      GOTO200                                                     C  0021016A12
156   SIGIN(NREAD)=(0.,0.)                                        C  0021016A15
200   CONTINUE                                                    C  0021016615
C           PRINT OUTPUT DATA SUMMARY IF NPRINT .GE. 3            C  0021016610
      IF(NPRINT-2)999,999,201                                     C  0021016610
201   WRITE(6,910)                                                C  0021017012
      WRITE(6,911)                                                C  0021017412
      IF(TIME.IS.INITE) GOTO202                                   C  0021017612
      WRITE(6,912)                                                C  0021017914
      GOTO205                                                     C  0021017012
202   WRITE(6,913)                                                C  0021017015
205   WRITE(6,914)RANGE                                           C  0021018212
      WRITE(6,915)FC,BW                                           C  0021018912
      WRITE(6,930)                                                C  0021019112
      WRITE(6,917)                                                C  0021019512
      WRITE(6,918)TO                                              C  0021019912
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      WRITE(6,920)NUMIN                                                    C  002101A012
      IF(NGAI)203,203,204                                                  C  002101A712
  203 WRITE(6,921)                                                         C  002101A013
  204 WRITE(6,922)LLIMIT                                                   C  002101AC12
      WRITE(6,923)NIMIN,NIMAX                                              C  002101B312
      WRITE(6,934)                                                         C  002101B012
      WRITE(6,935)GDELAY,RATTEN                                            C  002101BF12
      WRITE(6,936)                                                         C  002101C712
      WRITE(6,938)                                                         C  002101C812
      WRITE(6,939)                                                         C  002101CF12
      WRITE(6,940)                                                         C  002101D312
      DO206I=1,NPATH                                                       C  002101D712
      IN=T(I,1)+1.E-05                                                     C  002101D810
      WRITE(6,951)IN,T(I,2),T(I,3),T(I,5),T(I,6),T(I,7),T(I,8),T(I,9),T(   C  002101D815
     1I,10),T(I,11),T(I,12),T(I,13),T(I,14)                               C  002101F015
  206 CONTINUE                                                             C  002101FE12
          WRITE(6,930)                                                     C  002102D013
      WRITE(6,924)                                                         C  002102D412
      WRITE(6,930)                                                         C  002102D812
C         PRINT OUTPUT ARRAY = PRTOUT(N)                                   C  002102DC12
      WRITE(6,906)                                                         C  002102DC12
      NLPR=FLOAT(NPRINT)/3.                                                C  002102L012
      DO210L=1,NLPR                                                        C  002102L115
      LA=3*(L-1)+1                                                         C  002102L310
      LB=LA+1                                                              C  002102L511
      LC=LA+2                                                              C  002102L611
      WRITE(6,907)LA,PRTOUT(LA),LB,PRTOUT(LB),LC,PRTOUT(LC)                C  002102L714
  210 CONTINUE                                                             C  002102ZE12
C         END PRINTING OF OUTPUT DATA SUMMARY AND OUTPUT ARRAY             C  002102301 3
  900 FORMAT(A8)                                                           C  0021023013
  901 FORMAT(1X,'NPATH IS GREATER THAN 10; THIS EXCEEDS THE ALLOCATED AR   C  0021023013
     1RAY STORAGE. PROGRAM IS TERMINATED')                                 C  0021023013
  902 FORMAT(1X,'NCONVL IS GREATER THAN 400; THIS EXCEEDS THE ALLOCATED    C  0021023013
     1ARRAY STORAGE. PROGRAM IS TERMINATED')                               C  0021023013
  903 FORMAT(1X,'THE ARRAY STORAGE REQUIREMENTS FOR SIGIN(L) HAVE BEEN     C  0021023013
     1EXCEEDED. PROGRAM IS TERMINATED.')                                   C  0021023013
  904 FORMAT(2E13.6)                                                       C  0021023013
  905 FORMAT(1X,'THE ARRAY STORAGE REQUIREMENTS FOR PRTOUT(N) HAVE BEEN    C  0021023013
     1EXCEEDED. PROGRAM IS TERMINATED.')                                   C  0021023013
  906 FORMAT(5X,'INDEX',10X,'OUTPUT',17X,'INDEX',10X,'OUTPUT',17X,'INDEX   C  0021023013
     1',10X,'OUTPUT'//)                                                    C  0021023013
  907 FORMAT( 2X,I6, 2X,1P2E13.6,4X,I6, 2X,2E13.6, 4X,I6, 2X,2E13.6)       C  0021023013
  910 FORMAT(35X,'OUTPUT DATA SUMMARY'///)                                 C  0021023013
  911 FORMAT(1X,'IONOSPHERE-TRANSMITTER-RECEIVER PARAMETERS'/)             C  0021023013
  912 FORMAT(10X,'DAYTIME ELECTRON DENSITY PROFILE')                       C  0021023013
  913 FORMAT(10X,'NIGHTTIME ELECTRON DENSITY PROFILE')                     C  0021023013
  914 FORMAT(10X,'DISTANCE BETWEEN TRANSMITTER AND RECEIVER = ',F8.2,' K   C  0021023013
     1M.')                                                                 C  0021023013
  915 FORMAT(10X,'CARRIER FREQ. = ',F7.3,' MHZ.    BANDWIDTH = ',F7.2,' K  C  0021023013
     1HZ.')                                                                C  0021023013
  917 FORMAT(1X,'SIGNAL ANALYSIS PARAMETERS'/)                             C  0021023013
  918 FORMAT(10X,'THE SAMPLING INTERVAL,TO, EQUALS',1PE10.3,' SEC. (THIS   C  0021023013
     1 IS HALF THE NYQUIST SAMPLING INTERVAL)')                            C  0021023013
  920 FORMAT(10X,'THE LENGTH OF THE INPUT SEQUENCE IS ',I4)                C  0021023013
  921 FORMAT(10X,'THE GAINS FOR THE IONOSPHERE RETURNS ARE SET = 1 (THEY   C  0021023013
     1 ARE NONRANDOM)')                                                    C  0021023013
  922 FORMAT(10X,'THE IONOSPHERE IMPULSE RESPONSE IS TRUNCATED TO ',I3,'   C  0021023013
     1 SAMPLES')                                                           C  0021023013
  923 FORMAT(10X,'MINIMUM DELAY SHIFT = ',I4,3X,'MAXIMUM DELAY SHIFT = '   C  0021023013
     1,I4/)                                                                C  0021023013
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924  FORMAT(1X,'THE FOLLOWING DATA REPRESENTS THE RECEIVED, COMPLEX BAS         C  002102303
    1EBAND SIGNAL. IT INCLUDES THE GROUNDWAVE AND ALL IONOSPHERE RETURN         C  0021023013
    2S'//)                                                                       C  0021023013
930  FORMAT(1X)                                                                  C  0021023013
934  FORMAT(1X,'COMPUTER CHANNEL PARAMETERS'/)                                   C  0021023013
935  FORMAT(1X,'GROUNDWAVE:  DELAY = ',1PF10.3,' SEC.    ATTENUATION =          C  0021023013
    1',F10.3,' DB.')                                                             C  0021023013
936  FORMAT(1X,'IONOSPHERE RETURNS:'/)                                           C  0021023013
938  FORMAT( 3X,'MODE',3X,'SOLUTION',3X,'RAY ANGLE',3X,'PATH',4X,'CARRI        C  0021023013
    1ER',4X,'CARRIER',3X,'  SIGNAL  ',1X,'AMPLITUDE',4X,'PHASE',5X,'ATT         C  0021023013
    2ENUATION',2X,'DOPPLER',4X,'DOPPLER')                                        C  0021023013
939  FORMAT(10X,'INDICATOR',2X,'(DEGREES)',2X,'LENGTH',4X,'DELAY',6X,'P         C  0021023013
    1HASE',6X,'DELAY',3X,'DISTORTION',1X,' DISTORTION',5X,'(DB)', 7X,'S         C  0021023013
    2HIFT',5X,'SPREAD')                                                          C  0021023013
940  FORMAT( 9X,'(U=NU SOLN)',13X,'(KM)',5X,'(SEC)',4X,'(CYCLES)',5X,'(         C  0021023013
    1SEC)',4X,'(SEC/MHZ)',1X,'(SEC/MHZ**2)',15X,'(HZ)', 7X,'(HZ)'/)             C  0021023013
951  FORMAT(1X,I1,A2,1X,A4,4X,'NA',8X,F5.2,1X,F9.2,1X,1PE10.3,1X,E10.3,        C  0021023013
    11X,E10.3,1X,E10.3,2X,E10.3,2X,E10.3,1X,E10.3,1X,E10.3)                      C  0021023013
955  FORMAT(I1,A2,A4,4E16.9)                                                     C  0021023013
956  FORMAT(4E16.9)                                                              C  0021023013
957  FORMAT(3E16.9)                                                              C  0021023013
999  STOP                                                                        C  0021023013
     END                                                                         C  0021023112
     0021023313  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021005D
     0021023415  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210055
     0021023611  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021004D
     0021023713  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210045
     0021023815  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021003D
     0021023A11  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 00210033
     0021023B13  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0021001E
                                                               SEGMENT 002 IS 028B LONG
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                                                                         START OF SEGMENT 009
      SUBROUTINE IMPULS(NCONVL,NPATH,D2,D3,D)                          C  009:000010
      DIMENSION D2(10),D3(10)                                          C  009:000010
      COMPLEX D(10,801),XI(9),XI0,AI                                   C  009:000010
                                                                       C  009:000010
C         SUBROUTINE COMPUTES DELAY DISTORTION IMPULSE RESPONSES FOR   C  009:000010
C         EACH IONOSPHERE RETURN                                       C  009:000010
                                                                       C  009:000010
C             NCONVL= LENGTH OF TRUNCATED IMPULSE RESPONSE (.LE. 400)  C  009:000010
C             NPATH = NUMBER OF RETAINED IONOSPHERE RETURNS            C  009:000010
C             D2(I) = ARRAY OF AMPLITUDE DISTORTION COEFFICIENTS       C  009:000010
C             D3(I) = ARRAY OF PHASE DISTORTION COEFFICIENTS           C  009:000010
                                                                       C  009:000010
C         SUBROUTINE RETURNS ARRAY D(I,N). I DESIGNATES THE PARTICULAR C  009:000010
C         IONOSPHERE RETURN; N DESIGNATES THE ARGUMENT OF THE IMPULSE  C  009:000010
C         RESPONSE. N=1 CORRESPONDS TO AN ARGUMENT OF  -NCONVL*TO;     C  009:000010
C         N=2*NCONVL+1 CORRESPONDS TO AN ARGUMENT OF  +NCONVL*TO.      C  009:000010
                                                                       C  009:000010
C         WARNING: THE MAXIMUM ERROR IN THE FRESNEL INTEGRAL ( FROM    C  009:000010
C                  SUBROUTINE 'FRSNEL') IS 1.0 E-09. AS A CONSEQUENCE  C  009:000010
C                  THE ITERATIVE METHOD USED TO EVALUATE D(I,N) BREAKS C  009:000010
C                  DOWN WHEN N EXCEEDS APPROXIMATELY 25. FOR THIS      C  009:000010
C                  REASON IT IS NOT ADVISED TO CHOOSE NCONVL LARGER    C  009:000010
C                  THAN 25.                                            C  009:000010
                                                                       C  009:000010
      PI=3.14159265359                                                 C  009:000010
      AI=(0.,1.)                                                       C  009:000510
      NLIMIT=2*NCONVL+1                                                C  009:000611
      DO100I=1,NPATH                                                   C  009:000810
      DD2=D2(I)                                                        C  009:000910
      DD3=D3(I)                                                        C  009:000810
      DO50N=1,NLIMIT                                                   C  009:000010
      ARG=FLOAT(N-1-NCONVL)*PI                                         C  009:000E10
      IF(DD2)30,5,30                                                   C  009:001011
    5 IF(ARG)10,6,10                                                   C  009:001112
    6 XI0=(2.,0.)                                                      C  009:001213
      XI(1)=(0.,0.)                                                    C  009:001315
      XI(2)=(2./3.)*(1.,0.)                                            C  009:001512
      XI(3)=(0.,0.)                                                    C  009:001912
      XI(4)=(.4,0.)                                                    C  009:001810
      XI(5)=(0.,0.)                                                    C  009:001014
      XI(6)=(2./7.)*(1.,0.)                                            C  009:001F12
      XI(7)=(0.,0.)                                                    C  009:002313
      XI(8)=(2./9.)*(1.,0.)                                            C  009:002511
      XI(9)=(0.,0.)                                                    C  009:002912
      GOTO45                                                           C  009:002B10
   10 XI0=(2.,0.)*(SIN(ARG)/ARG)                                       C  009:002B13
      XI(1)=(-2.*AI*(COS(ARG)-(SIN(ARG)/ARG)))/ARG                     C  009:002F14
      DO20K=2,9                                                        C  009:003912
      K1=K-1                                                           C  009:003810
      XI(K)=(-AI/ARG)*((CEXP(AI*ARG)-((-1.)**FLOAT(K))*CEXP(-AI*ARG))+FL C  009:003C12
     1OAT(K)*XI(K1))                                                   C  009:004C13
   20 CONTINUE                                                         C  009:005311
      GOTO45                                                           C  009:005512
   30 AARG=ARG                                                         C  009:005515
      DDD2=DD2                                                         C  009:005614
      CALL FRSNEL(DDD2,AARG,XI0)                                       C  009:005713
      XI(1)=(-SIN(ARG)*CEXP(-AI*DD2)+.5*ARG*XI0)/DD2                   C  009:005915
      XI(2)=(AI*COS(ARG)*CEXP(-AI*DD2)+.5*ARG*XI(1)-AI*.5*XI0)/DD2     C  009:006911
      DO40K=3,9                                                        C  009:008111
```

AD-A054 726 SYSTEMS EXPLORATION INC SAN DIEGO CA F/G 17/2

HF CHANNEL SIMULATOR FOR WIDEBAND SIGNALS. A MATHEMATICAL MODEL--ETC(U)

MAR 78 R LUGANNANI, H C BOOKER, L E HOFF N00123-76-C-1090

UNCLASSIFIED NOSC-TR-208 NL

2 OF 2

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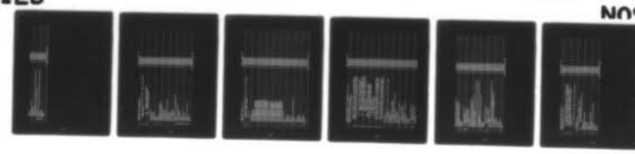

END

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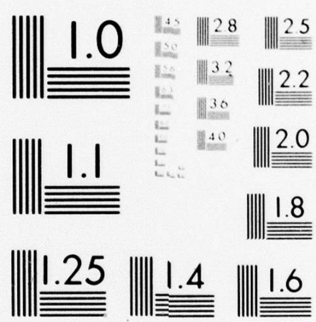

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

```
      K1=K-1                                                              C  0091008210
      K2=K+2                                                              C  0091008312
      KI=(FLOAT(K)/2.)+1.E-06                                             C  0091008415
      XINU=K-2*KI                                                         C  0091008715
      XINDC=1.-XIND                                                       C  0091008914
      XI(K)=(1./(2.*DD2))*(2.*AI*CEXP(-AI*DD2)*(XINDC*COS(ARG)+AI*XINU*S  C  00910008A15
     1IN(ARG))+ARG*XI(K1)-AI*FLOAT(K1)*XI(K2))                            C  0091009912
40    CONTINUE                                                            C  009100AC10
45    D(I,N)=.5*(XI0-AI*DD3*XI(3)-.5*(DD3**2)*XI(6)+AI*(1./6.)*(DD3**3)*  C  009100AE11
     1XI(9))                                                              C  009100C413
50    CONTINUE                                                            C  009100C615
100   CONTINUE                                                            C  009100CE10
      RETURN                                                              C  009100D011
      END                                                                 C  009100D014
                                                                    SEGMENT 009 IS 000D LONG
```

```
                                                                    START OF SEGMENT 00C
      SUBROUTINE FRSNEL(D2,ARG,XIO)                                 C  00C1000010
      COMPLEX AI,C2,C3,C4,C5,HD,HS,XIO                              C  00C1000010
                                                                    C  00C1000010
C         SUBROUTINE COMPUTES COMPLEX FRESNEL INTEGRAL USING RATIONAL C  00C1000010
C         APPROXIMATION.                                            C  00C1000010
                                                                    C  00C1000010
C            D2    = AMPLITUDE DISTORTION COEFFICIENT               C  00C1000010
C            ARG   = ARGUMENT OF FRESNEL INTEGRAL                   C  00C1000010
C            XIO   = COMPLEX FRESNEL INTEGRAL                       C  00C1000010
                                                                    C  00C1000010
      PI=3.14159265359                                              C  00C1000010
      AI=(0.,1.)                                                    C  00C1000510
      ARG=ABS(ARG)                                                  C  00C1000611
      IF(D2)1,35,2                                                  C  00C1000712
    1 SGN=-1.                                                       C  00C1000914
      GOTO3                                                         C  00C1000A13
    2 SGN=1.                                                        C  00C1000B10
    3 D2=ABS(D2)                                                    C  00C1000B14
      SUM=ABS(1.+(ARG/(2.*D2)))                                     C  00C1000C15
      DIFF=ABS(1.-(ARG/(2.*D2)))                                    C  00C1000F12
      C1=SQRT(PI/(2.*D2))                                           C  00C1001115
      C2=CEXP(-AI*D2)                                               C  00C1001412
      C3=CEXP(AI*ARG)                                               C  00C1001813
      C4=CEXP(-AI*ARG)                                              C  00C1001C10
      C5=CEXP((AI*(ARG**2))/(4.*D2))                                C  00C1002011
      ASUM=SUM/C1                                                   C  00C1002611
      ADIFF=DIFF/C1                                                 C  00C1002713
      W=ARG-2.*D2                                                   C  00C1002815
      CALL H(ADIFF,HD)                                              C  00C1002A14
      CALL H(ASUM,HS)                                               C  00C1002C13
      IF(W)5,10,10                                                  C  00C1002E12
    5 XIO=(1.-AI)*C1*C5-C1*C2*(HD*C3+HS*C4)                         C  00C1002F13
      GOTO20                                                        C  00C1004313
   10 XIO=C1*C2*(HD*C3-HS*C4)                                       C  00C1004410
   20 IF(SGN)25,25,30                                               C  00C1005013
   25 XIO=CONJG(XIO)                                                C  00C1005114
   30 CONTINUE                                                      C  00C1005312
      GOTO40                                                        C  00C1005312
   35 IF(ARG)37,36,37                                               C  00C1005315
   36 XIO=(2.,0.)                                                   C  00C1005510
      GOTO40                                                        C  00C1005612
   37 XIO=((2.,0.)*SIN(ARG))/ARG                                    C  00C1005615
   40 CONTINUE                                                      C  00C1005C11
      RETURN                                                        C  00C1005C11
      END                                                           C  00C1005C14
                                                            SEGMENT 00C IS 006A LONG
```

```
      SUBROUTINE H(W,HH)                                  START OF SEGMENT OOE
      COMPLEX A(12),C(12),AI,HH,S                         L   OOE:000010
                                                          L   OOE:000010
C         SUBROUTINE COMPUTES FUNCTION H(W) USED IN RATIONAL APPROX.  L   OOE:000010
C         OF COMPLEX FRESNEL INTEGRAL                     L   OOE:000010
                                                          L   OOE:000010
C         H(W) = HH                                       L   OOE:000010
                                                          L   OOE:000010
                                                          L   OOE:000010
      AI=(0.,1.)                                          L   OOE:000010
      PI=3.14159265359                                    L   OOE:000111
      E=(PI*(W**2))/2.                                    L   OOE:000610
      A(1)=(1.59576914,-.00000033)                        L   OOE:000715
      A(2)=(-.00001702,4.255387524)                       L   OOE:000C14
      A(3)=(-6.808568854,-.00009281)                      L   OOE:001113
      A(4)=(-.00057636,-7.7800204)                        L   OOE:001614
      A(5)=(6.920691902,-.00952095)                       L   OOE:001B14
      A(6)=(-.016898657,5.075161298)                      L   OOE:002014
      A(7)=(-3.050485660,-.138341947)                     L   OOE:002513
      A(8)=(-.075752419,-1.363729124)                     L   OOE:002A14
      A(9)=(.850663781,-.403340276)                       L   OOE:002F14
      A(10)=(-.025639041,.702222016)                      L   OOE:003414
      A(11)=(-.15023096,-.216105929)                      L   OOE:003913
      A(12)=(.034404779,.019547031)                       L   OOE:003E14
      C(1)=(0.,.19947114)                                 L   OOE:004313
      C(2)=(-.024933975,.00000023)                        L   OOE:004813
      C(3)=(.000003936,-.009351341)                       L   OOE:004D13
      C(4)=(.005770956,.000023006)                        L   OOE:005014
      C(5)=(.000689892,.004851466)                        L   OOE:005513
      C(6)=(-.009497136,.001903218)                       L   OOE:005A13
      C(7)=(.011948809,-.017122914)                       L   OOE:005F13
      C(8)=(-.006748873,.029064067)                       L   OOE:006414
      C(9)=(.00024642,-.027928955)                        L   OOE:006913
      C(10)=(.002102967,.016497308)                       L   OOE:006E14
      C(11)=(-.00121793,-.005508515)                      L   OOE:007313
      C(12)=(.000233939,.000838386)                       L   OOE:007814
      IF(E-4.)10,10,30                                    L   OOE:007D13
   10 Y=E/4.                                              L   OOE:007E15
      P=1.                                                L   OOE:008011
      S=(0.,0.)                                           L   OOE:008015
      DO20N=1,12                                          L   OOE:008210
      S=S+A(N)*P                                          L   OOE:008310
      P=P*Y                                               L   OOE:008614
   20 CONTINUE                                            L   OOE:008A10
      HH=.5*(1.-AI)*CEXP(AI*E)-S*SQRT(Y)                  L   OOE:008C11
      GOTO100                                             L   OOE:009613
   30 Y=4./E                                              L   OOE:009C10
      P=1.                                                L   OOE:009D12
      S=(0.,0.)                                           L   OOE:009E10
      DO40N=1,12                                          L   OOE:009F11
      S=S+C(N)*P                                          L   OOE:00A010
      P=P*Y                                               L   OOE:00A514
   40 CONTINUE                                            L   OOE:00A710
      HH=-S*SQRT(Y)                                       L   OOE:00A911
  100 RETURN                                              L   OOE:00AD14
      END                                                 L   OOE:00AE11
                                                       SEGMENT OOE IS 00BC LONG
```

```
                                                                   START OF SEGMENT 00F
      SUBROUTINE GAIN(ISEED,NCALL,BW,ALPHA,NPATH,T,C,X,Y,G)         C   00F1000010
      DIMENSION T(10,14)                                            C   00F1000010
      COMPLEX X(10,4),G(10),Y(10),R(10),AI                          C   00F1000010
      DOUBLE PRECISION C(10,7),D1,B1,D2,B2,DD2                      C   00F1000010
                                                                    C   00F1000010
C          SUBROUTINE COMPUTES RANDOM GAINS FOR IONOSPHERE RETURNS. ON C   00F1000010
C          EACH CALL THE ARRAY G(I) IS RETURNED. THE NUMBER OF ENTRIES C   00F1000010
C          IN G(I) = NPATH. THE ARRAY G(I) IS ASSOCIATED WITH THE SAMPLE C 00F1000010
C          TIME N.TO (N=0 FOR FIRST CALL, N=1 FOR SECOND CALL, ETC.).  C   00F1000010
                                                                    C   00F1000010
C              ISEED = SEED VALUE USED IN RANDOM NUMBER GENERATOR. IT  C   00F1000010
C                      SHOULD BE SET = 7**5 FOR FIRST CALL. IT IS      C   00F1000010
C                      REPLACED BY A NEW VALUE USED IN SUBSEQUENT CALLS. C 00F1000010
C              NCALL = NUMBER OF TIMES THE SUBROUTINE HAS BEEN CALLED. C   00F1000010
C                      NCALL SHOULD BE SET = 0 BEFORE FIRST CALL. IT IS C  00F1000010
C                      REPLACED BY A NEW VALUE USED IN SUBSEQUENT CALLS. C 00F1000010
C              BW    = BANDWIDTH (KHZ.)                                C   00F1000010
C              ALPHA = PARAMETER USED IN APPROXIMATE SPECTRUM          C   00F1000010
C              NPATH = NUMBER OF RETAINED IONOSPHERE RETURNS           C   00F1000010
C          THE ENTRIES OF T(I,J) USED IN THIS SUBROUTINE ARE:          C   00F1000010
C              I =  N: INDEX NUMBER FOR PATH (I .LE. NPATH)            C   00F1000010
C              J = 12: ATTENUATION (DB)                                C   00F1000010
C                = 13: DOPPLER SHIFT (HZ.)                             C   00F1000010
C                = 14: DOPPLER SPREAD (HZ.)                            C   00F1000010
C          THE ARRAYS C(I,J), X(I,J) AND Y(I) CONTAIN VARIABLES USED IN C  00F1000010
C          THE ALGORITHM. THEY SHOULD BE SET = 0 FOR FIRST CALL. THEY  C   00F1000010
C          ARE REPLACED BY NEW ARRAYS TO BE USED IN SUBSEQUENT CALLS.  C   00F1000010
                                                                    C   00F1000010
C          DOUBLE PRECISION ARITHMETIC IS USED FOR SOME CALCULATIONS TO C  00F1000010
C          OBTAIN THE DESIRED ACCURACY IN THE RANDOM GAINS. EARLIER    C   00F1000010
C          CALCULATIONS INDICATE THAT THE DOUBLE PRECISION IS ESSENTIAL. C 00F1000010
                                                                    C   00F1000010
      AI=(0.,1.)                                                    C   00F1000010
      PI=3.14159265359                                              C   00F1000111
C          THE SAMPLE INTERVAL TO IS HALF THE NYQUIST SAMPLE INTERVAL  C   00F1000610
      TO=1./(BW*2000.)                                              C   00F1000610
      IF(NCALL)10,10,21                                             C   00F1000715
   10 DO15I=1,NPATH                                                 C   00F1000910
      C(I,1)=DBLE(10.**(-T(I,12)/10.))                              C   00F1000A10
      C(I,2)=DBLE(PI*ALPHA*T(I,14)*TO)                              C   00F1000F14
      C(I,3)=DSQRT((4.*C(I,1)/3.))*C(I,2)**2.5)                     C   00F1001510
      C(I,4)=DEXP(-C(I,2))                                          C   00F1001E13
      C(I,5)=C(I,4)**2                                              C   00F1002310
      C(I,6)=C(I,4)**3                                              C   00F1002811
      C(I,7)=DBLE(2.*PI*T(I,13)*TO)                                 C   00F1002D12
   15 CONTINUE                                                      C   00F1003214
C          COMPUTE G(I) FOR N=0                                     C   00F1003415
      CALL RANDOM(ISEED,NPATH,R)                                    C   00F1003415
      DO20I=1,NPATH                                                 C   00F1003711
      X(I,1)=SQRT(SNGL(C(I,1)))*R(I)                                C   00F1003B10
      G(I)=X(I,1)                                                   C   00F1003F13
   20 CONTINUE                                                      C   00F1004215
      NCALL=1                                                       C   00F1004510
      GOTO100                                                       C   00F1004514
C          END COMPUTATION FOR N=0                                  C   00F1004611
   21 IF(NCALL-1)25,25,31                                           C   00F1004611
C          COMPUTE G(I) FOR N=1                                     C   00F1004715
   25 CALL RANDOM(ISEED,NPATH,R)                                    C   00F1004715
      DO30I=1,NPATH                                                 C   00F1004A11
```

```
      D1=(1.+C(I,2)+(1./3.)*(C(I,2)**2))*C(I,4)                    C   OUF1004610
      R1=DSQRT(C(I,1))*DSQRT(1.-(D1**2))                             C   OUF1005313
      X(I,2)=SNGL(R1)*R(I)+SNGL(D1)*X(I,1)                           C   OUF1005911
      G(I)=CEXP(AI*SNGL(C(I,7)))*X(I,2)                              C   OUF1006411
   30 CONTINUE                                                       C   OUF1006611
      NCALL=2                                                        C   OUF1007012
      GOTO100                                                        C   OUF1007111
C           END COMPUTATION FOR N=1                                  C   OUF1007114
   31 IF(NCALL-2)35,35,41                                            C   OUF1007214
C           COMPUTE G(I) FOR N=2                                     C   OUF1007313
   35 CALL RANDOM(ISEED,NPATH,R)                                     C   OUF1007313
      CALL RANDOM(ISEED,NPATH,Y)                                     C   OUF1007515
      DO40I=1,NPATH                                                  C   OUF1007612
      D1=(1.+C(I,2)+(1./3.)*(C(I,2)**2))*C(I,4)                      C   OUF1007910
      R1=DSQRT(C(I,1))*DSQRT(1.-(D1**2))                             C   OUF1008113
      DD2=(1.+2.*C(I,2)+(4./3.)*(C(I,2)**2))*C(I,5)                  C   OUF1008711
      D2=(DD2-D1**2)/(1.-D1**2)                                      C   OUF1009012
      R2=DSQRT(C(I,1))*DSQRT( (1.+D2)*(1.-DD2)-(4./3.)*(C(I,2)**5)*C(I,5
     1))                                                             C   OUF1009511
      X(I,3)=SNGL(R2)*R(I)+SNGL(C(I,3)*C(I,4))*Y(I)+SNGL(D1*(1.-D2))*X(I
     1,2)+SNGL(D2)*X(I,1)                                            C   OUF100A612
      G(I)=CEXP(AI*SNGL(C(I,7))*FLOAT(NCALL))*X(I,3)                 C   OUF100B915
   40 CONTINUE                                                       C   OUF100C513
      NCALL=3                                                        C   OUF100C714
      GOTO100                                                        C   OUF100C813
C           END COMPUTATION FOR N=2                                  C   OUF100C910
                                                                     C   OUF100C910
C           COMPUTE G(I) FOR N.GE.3                                  C   OUF100C910
   41 CALL RANDOM(ISEED,NPATH,R)                                     C   OUF100C910
      DO90I=1,NPATH                                                  C   OUF100C812
      X(I,4)=SNGL(C(I,3)*C(I,4))*(R(I)+SNGL(C(I,4))*Y(I))+SNGL(C(I,6))*X C   OUF100CC10
     1(I,1)-3.*SNGL(C(I,5))*X(I,2)+3.*SNGL(C(I,4))*X(I,3)            C   OUF100D615
      G(I)=CEXP(AI*SNGL(C(I,7))*FLOAT(NCALL))*X(I,4)                 C   OUF100F013
      X(I,1)=X(I,2)                                                  C   OUF100FC11
      X(I,2)=X(I,3)                                                  C   OUF100FF14
      X(I,3)=X(I,4)                                                  C   OUF1010312
      Y(I)=R(I)                                                      C   OUF1010710
   90 CONTINUE                                                       C   OUF1010915
      NCALL=NCALL+1                                                  C   OUF1010C10
  100 RETURN                                                         C   OUF1010D12
      END                                                            C   OUF1010D15
                                                        SEGMENT OUF IS 011F LONG
```

```
                                                                     START OF SEGMENT 013
      SUBROUTINE RANDOM(ISEED,NPATH,R)                               C  013:000010
      COMPLEX R(10),AI                                               C  013:000010
      DOUBLE PRECISION D2P31M,D2PN31,Z                               C  013:000010
                                                                     C  013:000010
C          THIS SUBROUTINE RETURNS THE ARRAY R(I); EACH ENTRY OF R(I)  C  013:000010
C          IS A COMPLEX GAUSSIAN VARIABLE; THE REAL AND IMAGINARY PARTS C  013:000010
C          ARE INDEPENDENT, IDENTICALLY-DISTRIBUTED, ZERO-MEAN, UNIT-  C  013:000010
C          VARIANCE GAUSSIAN VARIABLES. ALL ENTRIES OF R(I) ARE        C  013:000010
C          INDEPENDENT AND ARE INDEPENDENT OF THE ARRAY PRODUCED      C  013:000010
C          ON PREVIOUS CALLS                                         C  013:000010
                                                                     C  013:000010
C          ISEED IS REPLACED BY A NEW ISEED TO BE USED IN SUBSEQ. CALLS C  013:000010
                                                                     C  013:000010
      AI=(0.,1.)                                                     C  013:000010
      TPI=6.283185307180                                             C  013:000111
C          D2P31M = (2**31)-1; D2PN31 = 1/(2**31)                    C  013:000010
      DATA        D2P31M/2147483647.D0/,                             C  013:000010
     *            D2PN31/01274000000000000000000000000000/           C  013:000010
      Z=ISEED                                                        C  013:000010
      DO50L=1,NPATH                                                  C  013:000710
    1 Z=DMOD(16807.D0*Z,D2P31M)                                      C  013:000010
      U1=Z*D2PN31                                                    C  013:000A11
      IF(U1)3,3,2                                                    C  013:000012
    3 Z=.57706925                                                    C  013:000011
      GOTO1                                                          C  013:000014
    2 Z=DMOD(16807.D0*Z,D2P31M)                                      C  013:000011
      U2=Z*D2PN31                                                    C  013:001112
      R(L)=SQRT(-2.*ALOG(U1))*CEXP(AI*TPI*U2)                        C  013:001213
   50 CONTINUE                                                       C  013:001014
      ISEED=Z                                                        C  013:001115
      RETURN                                                         C  013:002015
      END                                                            C  013:002112
                                                                SEGMENT 013 IS 0029 LONG
```

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